A Guide to Exponents

Teaching Approach

These lessons are designed to develop learners basic understanding and problem-solving and cognitive skills. We introduce the basic concepts first in each lesson and then build on this knowledge through application.

The introductory video is designed to show learners practical applications of exponents and to reinforce the concept of writing exponents as repeated multiplication. In order to appreciate the numerical values of powers, your learners need to experience the incredible rate of growth that is produced by repeated multiplication. The chessboard example provides an excellent indication of the power of exponential growth.

Remind learners about the laws for multiplying and dividing exponents from grade 9. Explain the laws and the restrictions on the bases. It is important to include examples with coefficients when dividing exponential expressions, so that they learn not to confuse the rules for dividing numbers with dividing exponents. Learners must say the laws out loud and commit them to memory.

Make sure you reinforce the link between dividing and negative exponents. Revise what it means if a base is written with no exponent, e.g. what is the exponent of x? If we do not write an exponent, it does not mean that the number has no exponent; the exponent is 1.

Before starting the lesson on raising a power to a power, emphasise what it means when we use a bracket in algebra: e.g. $(2^2)^3$ means that everything inside the bracket is being raised to the power of three. Make sure that your learners write the examples as repeated multiplication. Do examples that make use of coefficients inside the bracket as well.

Make sure that learners can state the laws very specifically, including statements such as ‘if the bases are the same’. The game show activity in the lesson on applying the laws of exponents lends itself to dividing the class into pairs or teams. You might want to set up a real quiz situation, and reward the winning team. Alternatively, you could use this activity for individual assessment. You could ask learners to explain their answers.

Many learners confuse exponents that are fractions with numbers that are fractions. Use different bases and many examples to explain the difference.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson.

1 Introduction to Exponents

In this video we see real life applications of exponential growth. We also write repeated multiplication as numbers in exponential form.

2 Multiplying and Dividing Exponents

This video shows how to simplify expressions using the laws for multiplication and division of powers for integral exponents.

3 Negative and Zero Exponents

The Negative and Zero Exponent Video shows how to convert powers with a negative exponent to powers with a positive exponent and explain the meaning of a power with zero as exponent. We highlight the connection between division and negative powers.

4 Raising a Power to a Power

In this video, viewers are shown how to simplify expressions by raising a power to a power. The video also explains the importance of raising every exponent inside the bracket to the exponent outside the bracket.

5 Applying the Laws of Exponents

This lesson can be used as a revision of the laws of exponents. Sections of it are done in a game show format, giving the viewer a chance to test their skills. It covers simplifying expressions using the laws of exponents for integral exponents.

6 Prime Factorisation of Bases

Prime factorisation is a skill that is taught in lower grades but used extensively in this section. This video revises the process and shows the importance of finding the prime factors of bases in problems.

7 Exponents with Fractions

In this lesson we calculate with powers that have rational exponents. We also convert from surd form to exponential form.

8 Factorising Exponential Expressions

Here we focus on exponential expressions that involve addition and subtraction and have to be factorised, in order to be simplified. We look at factorising exponential trinomials.

9 Using all the Exponential Laws

This lesson reviews all the laws, including rational exponents, to simplify exponential expressions.

10 Exponential Equations I

Learners need to be able to prime factorise before attempting this section. In this lesson the variable is only in the exponent and the bases can be simplified to equivalent numbers.

11 Exponential Equations II

Here we use the method of trial and error to solve an exponential equation with different bases. The method of interval bisection is used to solve these equations. We also solve exponential trinomials.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1 Introduction to Exponents	http://www.winpossible.com/lessons/Algebra_I_Getting_started_-_Exponent_and_Root_Rules.html	Getting started with exponents and roots.
	http://www.math.com/school/subject2/lessons/S2U2L2GL.html	Exponents:
2 Multiplying and Dividing Exponents	http://www.winpossible.com/lessons/Algebra_I_Multiplying_and_dividing_two_numbers_with_same_base_and_different_exponents.html	Multiply and divide numbers with the same base and different exponents.
	http://www.purplemath.com/modules/simpexpo.htm	Simplifying Expressions with Exponents.
3 Negative and Zero Exponents	http://www.winpossible.com/lessons/Algebra_I_Calculating_and_working_with_zero_exponents.html	Calculate and work with zero exponents.
	http://www.college-cram.com/study/algebra/basic-pre-algebra/properties-of-exponents/	Properties of Exponents
4 Raising a Power to a Power	http://www.math.com/school/subject2/lessons/S2U2L2EX.html	A few common errors students make when working with exponents.
	http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_ExponentsRules.xml	Rules of Exponents.
5 Applying the Laws of Exponents	http://www.purplemath.com/modules/simpexpo.htm	Simplifying Expressions with Exponents.
6 Prime Factorisation of Bases	http://mathforum.org/isaac/problems/prime1.html	Prime numbers.
7 Exponents as Fractions	http://www.college-cram.com/study/algebra/basic-pre-algebra/rational-exponents/	Rational exponents.
8 Factorising Exponential Expressions	http://www.mathway.com/problem.aspx?p=basicmath	Enter math questions and get them solved online.
9 Using all Exponential Laws	http://www.mathsisfun.com/algebra/exponent-laws.html	Laws of Exponents.
10 Exponential Equations 1	http://everythingmaths.co.za/grade-10/03-exponentials/03-exponentials-03.cnxmlplus	Exponential equations (textbook).
11 Exponential Equations 2	http://everythingmaths.co.za/grade-10/03-exponentials/03-exponentials-03.cnxmlplus	Exponential equations (textbook).
	http://exchange.smarttech.com/search.html?q=exponential%20function	Lessons for your Smart board.

Task

Question 1

Your parents ask you to wash the dishes. You agree but ask them to pay you 2 cents on the first night, 4 cents on the second night, 8cents on the third night and continue like this. Use this table to work out what you'll be paid on the 30th night.

Night	Amount	As a power
1	2c	2^1
2	4c	2^2
3	8c	2^3
10		
30		

Question 2

State whether the following are true or false and give a reason for your answer.

2.1 $a^2 \times b^3 = (ab)^{2+3}$

2.2 $\dfrac{1}{2ab^{-3}} = 2ab^3$

Question 3

Simplify without the use of a calculator. Write your answers with positive exponents:

3.1 $(9x^2y^0)^{-2} \times 81y^{-2}x^5$

3.2 $\dfrac{(45)^y.25^{y-2}}{3^y.125}$

3.3 $(8p^3)^{\frac{1}{3}} \times (4p^{-2})^{\frac{1}{2}}$

Question 4

Factorise: $\dfrac{5.3^x - 9.3^{x-2}}{3^x - 3^{x-1}}$

Question 5

5.1 Simplify: $\dfrac{3^{2x}-1}{3^x+1}$

5.2 Hence solve:

$$\frac{3^{2x}-1}{3^x+1} = 26$$

Question 6

Solve for x: $2\left(\dfrac{1}{2}\right)^{-x} = 5^{-2}.5^2$

Question 7

Solve by trial and error: $2^x = 5$

Question 8

Factorise and solve for x:

$x^{\frac{1}{2}} - 6x^{\frac{1}{4}} - 27 = 0$

Task Answers

Question 1

Night	Amount	As a power
1	2c	2^1
2	4c	2^2
3	8c	2^3
10		
30		

On night 30 you get $2^{30} = 1073741824$ (parent will be bankrupt long before this!)

Question 2

2.1 False. Bases are different so we cannot add the powers.

2.2 False. Only the 'b' should move to make the power positive, the 2 and a must stay in the denominator.

Question 3

3.1

$$(9x^2y^0)^{-2} \times 81y^{-2}x^5 = (3^2)^{-2}x^{-4}y^0 \times 3^4 y^{-2}x^5$$
$$= 3^{-4+4}x^{-4+5}y^{-2}$$
$$= \frac{3^0 x^1}{y^2}$$
$$= \frac{x}{y^2}$$

3.2

$$\frac{(45)^y.25^{y-2}}{3^y.125}$$
$$= \frac{(3^2 \times 5)^y.(5^2)^{y-2}}{3^y.5^3}$$
$$= \frac{3^{2y}.5^y.5^{2y-4}}{3^y.5^3}$$
$$= 3^{2y-y}.5^{y+2y-4-3}$$
$$= 3^y.5^{3y-7}$$

3.3

$$(8p^3)^{\frac{1}{3}} \times (4p^{-2})^{\frac{1}{2}}$$
$$= (2^3 p^3)^{\frac{1}{3}} \times (2^2 p^{-2})^{\frac{1}{2}}$$
$$= (2^3)^{\frac{1}{3}} (p^3)^{\frac{1}{3}} \times (2^2)^{\frac{1}{2}} (p^{-2})^{\frac{1}{2}}$$
$$= 2p \times 2p^{-1}$$
$$= 4p^0$$
$$= 4$$

Question 4

$$\frac{5.3^x - 9.3^{x-2}}{3^x - 3^{x-1}}$$
$$= \frac{5.3^x - 9.3^x.3^{-2}}{3^x - 3^x.3^{-1}}$$
$$= \frac{3^x(5 - 9.3^{-2})}{3^x(1 - 3^{-1})}$$
$$= \frac{5 - 9(\frac{1}{9})}{1 - \frac{1}{3}} = 6$$

Question 5

5.1

$$\frac{3^{2x} - 1}{3^x + 1}$$
$$= \frac{(3^x - 1)(3^x + 1)}{3^x + 1}$$
$$= 3^x - 1$$

5.2

$$\frac{3^{2x} - 1}{3^x + 1} = 26$$
$$\frac{(3^x - 1)(3^x + 1)}{3^x + 1} = 26$$
$$3^x - 1 = 26$$
$$3^x = 27$$
$$3^x = 3^3$$
$$\therefore x = 3$$

Question 6

$$2\left(\frac{1}{2}\right)^{-x} = 5^{-2}.5^{2}$$

$$(\frac{1}{2})^{-x} = \frac{1}{2}$$

$$-x = 1$$

$$\therefore x = -1$$

Question 7

$$2^{x} = 5$$

$$2^{2} < 2^{x} < 2^{3}$$

$$\therefore 2 < x < 3$$

$$x \approx \frac{2+3}{2} = 2.5$$

$$2^{2.5} = 5.65685....$$

$$\therefore 2 < x < 2.5$$

$$x \approx \frac{2+2.5}{2} = 2.25$$

$$2^{2.25} = 4.75682...$$

$$\therefore 2.25 < x < 2.5$$

$$x \approx \frac{2.25+2.5}{2} = 2.375$$

$$2^{2.375} = 5.187...$$

$$\therefore 2.25 < x < 2.375$$

$$x \approx \frac{2.25+2.375}{2} = 2.3125$$

$$2^{2.3125} = 4.967$$

$$2^{2.31} = 4.958...and\,2^{2.32} = 4.99...$$

$$\therefore x \approx 2.32$$

Question 8

$$x^{\frac{1}{2}} - 6x^{\frac{1}{4}} - 27 = 0$$

$$(x^{\frac{1}{4}})^{2} - 6x^{\frac{1}{4}} - 27 = 0$$

$$(x^{\frac{1}{4}} - 9)(x^{\frac{1}{4}} + 3) = 0$$

$x^{\frac{1}{4}} + 3 = 0$ ….. no solution

$$x^{\frac{1}{4}} = 9$$

$$(x^{\frac{1}{4}})^{4} = (3^{2})^{4}$$

$$\therefore x = 3^{8} = 6561$$

Acknowledgements

Mindset Learn Executive Head	Dylan Busa
Content Manager Classroom Resources	Jenny Lamont
Content Coordinator Classroom Resources	Helen Robertson
Content Administrator	Agness Munthali
Content Developer	Natashia Bearam
Content Reviewer	Malindri Eastes
	Helen Robertson

Produced for Mindset Learn by Traffic

Facilities Coordinator	Cezanne Scheepers
Production Manager	Belinda Renney
Director	Alriette Gibbs
Editor	Nonhlanhla Nxumalo
Presenter	Lebo Mazibuko
Studio Crew	Abram Tjale
	James Tselapedi
Graphics	Wayne Sanderson