

# *A Guide to Exponential and Logarithmic Functions*

## **Teaching Approach**

Exponents and logarithms are covered in the first term of Grade 12 over a period of one week. We cover the laws of exponents and laws of logarithms. The relation between the exponential and logarithmic graph is explored. The reflection in the line  $y = x$  is used to make it easy for learners to identify key points like the y intercept and the asymptote

Summaries of skills and context of each video are in this document, allowing you to find something appropriate, quickly and easily.

Each video is short enough to fit into a lesson with time to discuss the content and do some related work. You will find a selection of tasks covering the required skills in the task video. These tasks have not been linked to the videos so that they can be used without viewing them.

When teaching learners exponents and logarithms to learners it is important that you give learners work in different context. Help learners sketch and interpret graphs. Use colour to highlight important points like intercepts and asymptotes. Make sure that you revise the laws of exponents and surds to help your learners.

## Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

### 1. Revising Exponents and Surds

In this video, we discuss the laws of exponents and how they are used in simplifying and solving problems involving exponents. We also look how we rationalise surds.

### 2. Revising the Exponential Function

We discuss the characteristics of an exponential function. We look at the variations that are due to reflections and translations. We also look at the value of the base of the exponent and how that affects the shape of the graph.

### 3. Exploring the Exponential Function

We discuss the effect of  $a$  on the  $y$  - intercept, the asymptote and the shape in general. We also look at how  $q$  affects the asymptote of the exponential graph.

### 4. Introducing the Laws of Logarithms

In the video we define a logarithm. We also discuss the laws of logarithms and how logarithms relate to exponents. We solve exponential equations using the logarithms and vice versa.

### 5. Logarithmic Functions

In this video, we discuss how the logarithmic function relates to the exponential function. We extensively use the reflection in the line  $y=x$ . We explore the asymptote and the  $x$  - intercept. We also touch the effect of the value of the base on the shape of the graph.

## Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1. Revising Exponents and Surds	<a href="http://booki.cc/mathematics-11-2/01-exponents-and-surds/">http://booki.cc/mathematics-11-2/01-exponents-and-surds/</a>	Notes on exponents and surds
	<a href="http://everythingmaths.co.za/grade-11/01-exponents-and-surds/01-exponents-and-surds-03.cnxmplus">http://everythingmaths.co.za/grade-11/01-exponents-and-surds/01-exponents-and-surds-03.cnxmplus</a>	Solving surd equations
	<a href="http://m.everythingmaths.co.za/grade-11/01-exponents-and-surds/01-exponents-and-surds-02.cnxmplus">http://m.everythingmaths.co.za/grade-11/01-exponents-and-surds/01-exponents-and-surds-02.cnxmplus</a>	Rational Exponents and surds
2. Revising the Exponential Function	<a href="http://en.wikipedia.org/wiki/Exponential_function">http://en.wikipedia.org/wiki/Exponential_function</a>	Definition of an exponential function
	<a href="http://www.purplemath.com/modules/expofcns.htm">http://www.purplemath.com/modules/expofcns.htm</a>	Introduction to exponential functions
	<a href="http://www.freemathhelp.com/exponential-functions.html">http://www.freemathhelp.com/exponential-functions.html</a>	Free math Help lessons
3. Exploring the Exponential Function	<a href="http://fym.la.asu.edu/~tturner/MAT_117_online/Exponential_Functions/Exponentials.htm">http://fym.la.asu.edu/~tturner/MAT_117_online/Exponential_Functions/Exponentials.htm</a>	Basic features of an exponential function
	<a href="http://www.youtube.com/watch%3Fv%3D6Db1wG_zudc">http://www.youtube.com/watch%3Fv%3D6Db1wG_zudc</a>	Video on transforming exponential functions
	<a href="http://www.mrl.ucsb.edu/sites/default/files/mrl_docs/ret_attachments/curriculum/Kirk_handou">http://www.mrl.ucsb.edu/sites/default/files/mrl_docs/ret_attachments/curriculum/Kirk_handou</a>	Notes on exponential functions
4. Introducing the Laws of Logarithms	<a href="http://calculus.nipissingu.ca/tutorials/logarithms.html">http://calculus.nipissingu.ca/tutorials/logarithms.html</a>	Tutorial on logarithms
	<a href="http://whatis.techtarget.com/definition/logarithm-logarithmic">http://whatis.techtarget.com/definition/logarithm-logarithmic</a>	Definition of logarithms
	<a href="http://www.bbc.co.uk/bitesize/higher/maths/algebra/logarithms/revision/3/">http://www.bbc.co.uk/bitesize/higher/maths/algebra/logarithms/revision/3/</a>	Application of logarithms
5. Logarithmic Functions	<a href="http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut43_logfun.htm">http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut43_logfun.htm</a>	Tutorial on logarithmic functions
	<a href="http://www.shoreline.edu/math-learning-center/documents/properties-of-logarithms.pdf">http://www.shoreline.edu/math-learning-center/documents/properties-of-logarithms.pdf</a>	Properties of logarithmic functions
	<a href="http://www.themathpage.com/aprecalc/logarithmic-exponential-functions.htm">http://www.themathpage.com/aprecalc/logarithmic-exponential-functions.htm</a>	Relation between logarithmic and exponential functions

**Task**
**Question 1**

Solve the following equations:

1.1  $\log_2 x = 8$

1.2  $\log_5 125 = m$

1.3  $\log_b 81 = 4$

1.4  $27 = 3^x$

1.5  $32 = x^5$

1.6  $128 = 4^x$

1.7  $5 = \log_2 8x$

**Question 2**

Simplify the following:

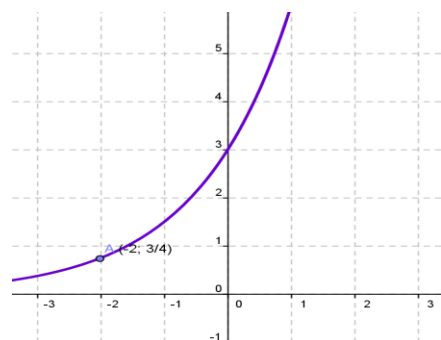
2.1  $\frac{3^{x+1} + 3^{x+2}}{3^{x-2}}$

2.2  $\frac{5^{2x+3} - 25^{x-1}}{5^{2x+1}}$

2.3  $\frac{2^{3x+1} - 8^{x+2}}{2^{3x+5} - 2^{3x}}$

**Question 3**

The graph of  $m$  passes through the point  $(-2; \frac{3}{4})$  and has a  $y$  - intercept at  $(0; 3)$



3.1 If  $m(x) = a \cdot b^x$ , find the values of  $a$  and  $b$

 3.2 Determine  $n(x)$  if  $n$  is the inverse of the function  $m$ 

 3.3 Sketch the graph of  $n(x)$ 

 3.4 Find the  $x$  - intercept for  $n(x - 3)$ 

 3.5 For which values of  $x$  is  $n(x) < 0$ 

 3.6 What is the asymptote of  $m(x) + 4$ 
**Question 4**

Consider the function  $f(x) = 3^{2x} - 3$

 4.1 Calculate the  $x$  - intercept

 4.2 Calculate the  $y$  - intercept

 4.3 Write down the equation of the asymptote of  $f$ .

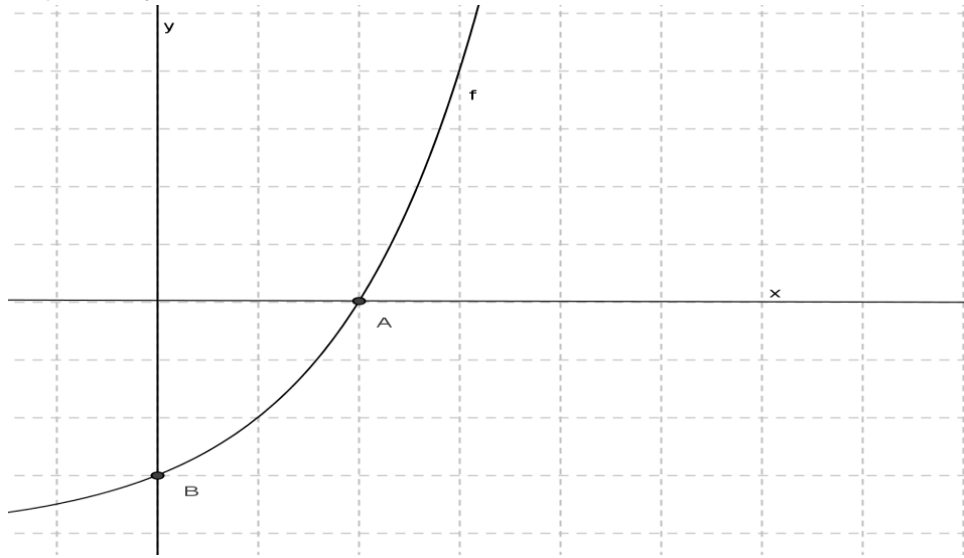
 4.4 Sketch the graph of  $f$ 

 4.5 Write down the equation of  $g(x)$ , if  $g$  is the reflection of  $f$  about the  $x$ -axis

 4.6 Solve for  $x$  if  $f(x) = 6$

**Question 5**

The graph of  $f(x) = 2^x - 4$  is sketched below. A and B are the  $x$  - and  $y$  - intercepts respectively.



- 5.1 Determine the coordinates of A and B
- 5.2 Determine the equation of  $g(x)$  the inverse of  $f(x) + 4$
- 5.3 Write down the equation of the asymptote of  $g$
- 5.4 Determine the equation of  $h$  if  $h(x) = f(-x) + 4$
- 5.5 Describe the transformation that maps  $f$  onto  $h$

## Task Answers

### Question 1

1.1  $\log_2 x = 8$

$$2^8 = x$$

$$x = 256$$

1.2  $\log_5 125 = m$

$$5^m = 125$$

$$5^m = 5^3$$

$$\therefore m = 3$$

1.3  $\log_b 81 = 4$

$$b^4 = 81$$

$$b^4 = 3^4$$

$$\therefore b = 3$$

1.4  $27 = 3^x$

$$3^3 = 3^x$$

$$\therefore x = 3$$

1.5  $32 = x^5$

$$2^5 = x^5$$

$$\therefore x = 2$$

1.6  $128 = 4^x$

$$2^7 = 2^{2x}$$

$$2x = 7$$

$$x = 3\frac{1}{2}$$

1.7  $5 = \log_2 8x$

$$2^5 = 8x$$

$$32 = 8x$$

$$x = 4$$

### Question 2

2.1  $\frac{3^{x+1} + 3^{x+2}}{3^{x-2}}$

$$= \frac{3^x \cdot 3^1 + 3^x \cdot 3^2}{3^x \cdot 3^{-2}}$$

$$= \frac{3^x(3^1 + 3^2)}{3^x(3^{-2})}$$

$$= \frac{3 + 9}{\frac{1}{9}}$$

$$= 12 \times 9$$

$$= 108$$

$$\begin{aligned}
 2.2 \quad & \frac{5^{2x+3} - 25^{x+1}}{5^{2x+1}} \\
 &= \frac{5^{2x} \cdot 5^3 - 5^{2x} \cdot 5^2}{5^{2x} \cdot 5^1} \\
 &= \frac{5^{2x}(5^3 - 5^2)}{5^{2x} \cdot 5^1} \\
 &= \frac{125 - 25}{5} \\
 &= \frac{100}{5} \\
 &= 20
 \end{aligned}$$

$$\begin{aligned}
 2.3 \quad & \frac{2^{3x+1} - 8^{x+2}}{2^{3x+5} - 2^{3x}} \\
 &= \frac{2^{3x} \cdot 2^1 - 2^{3x} \cdot 2^6}{2^{3x} \cdot 2^5 - 2^{3x} \cdot 2^0} \\
 &= \frac{2^{3x}(2^1 - 2^6)}{2^{3x}(2^5 - 1)} \\
 &= \frac{2 - 64}{32 - 1} \\
 &= \frac{-62}{31} \\
 &= -2
 \end{aligned}$$

### Question 3

3.1 when  $x = 0$   $y = 3$

$$a. b^x = m(x)$$

$$a. b^0 = 3$$

$$a. 1 = 3$$

$$a = 3$$

When  $x = -2$   $y = \frac{3}{4}$

$$3. b^{-2} = \frac{3}{4}$$

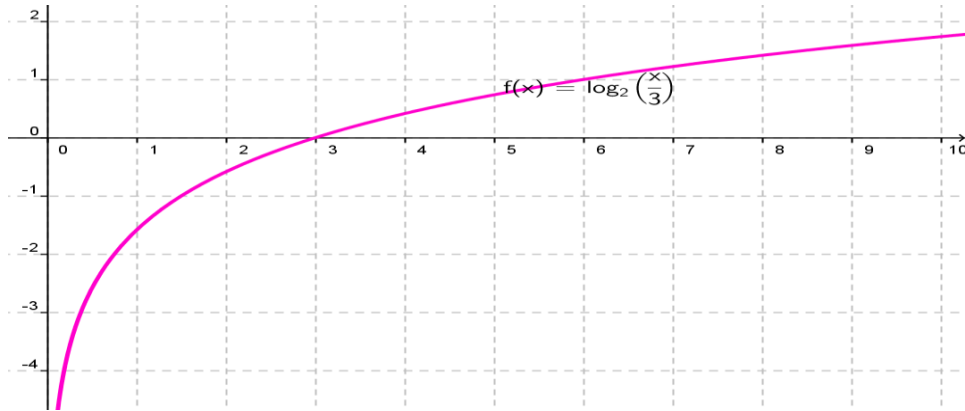
$$b^{-2} = \frac{1}{4}$$

$$b^{-2} = 2^{-2}$$

$$\therefore b = 2$$

3.2  $m(x) = 3 \cdot 2^x$   
 $x = 3 \cdot 2^y$   
 $\frac{x}{3} = 2^y$   
 $n(x) = \log_2 \frac{x}{3}$

3.3



3.4  $n(x - 3)$  is a horizontal shift of 3 units to the right, therefore the  $x$  intercept of  $n(x - 3)$  will be  $(6; 0)$

3.5 From the graph  $0 < x < 3$

3.6 The asymptote is  $y = 4$

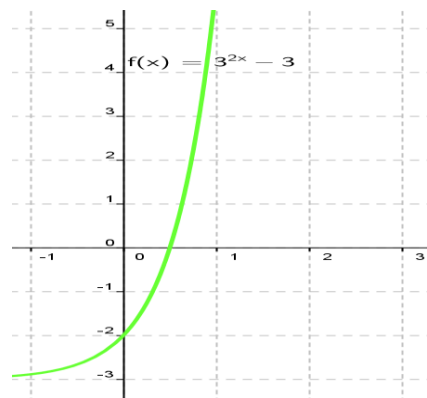
**Question 4**

4.1  $3^{2x} - 3 = 0$   
 $3^{2x} = 3$   
 $3^{2x} = 3^1$   
 $2x = 1$   
 $x = \frac{1}{2}$

4.2  $y = 3^{2x} - 3$   
 $y = 3^0 - 3$   
 $y = 1 - 3$   
 $y = -2$

4.3  $y = -3$

4.4





$$\begin{aligned}
 4.5 \quad g(x) &= -f(x) \\
 &= -(3^{2x} - 3) \\
 &= -3^{2x} + 3
 \end{aligned}$$

$$\begin{aligned}
 4.6 \quad 3^{2x} - 3 &= 6 \\
 3^{2x} &= 9 \\
 3^{2x} &= 3^2 \\
 2x &= 2 \\
 x &= 1
 \end{aligned}$$

**Question 5**

$$\begin{aligned}
 5.1 \quad 2^x - 4 &= 0 \\
 2^x &= 4 \\
 2^x &= 2^2 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 &A(2;0) \\
 f(x) &= 2^0 - 4 \\
 f(x) &= 1 - 4 \\
 f(x) &= -3 \\
 &B(0; -3)
 \end{aligned}$$

$$\begin{aligned}
 5.2 \quad f(x) + 4 &= 2^x - 4 + 4 \\
 &= 2^x
 \end{aligned}$$

$$\begin{aligned}
 x &= 2^y \quad y = \log_2 x \\
 g(x) &= \log_2 x \\
 x &= 0
 \end{aligned}$$

$$\begin{aligned}
 5.3 \quad h(x) &= 2^{-x} - 4 + 4 \\
 h(x) &= 2^{-x}
 \end{aligned}$$

5.4  $h$  is mapped by a vertical translation of 4 units up followed by a reflection in the  $y$ -axis (or  $x = 0$ )

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