A Guide to Sine, Cosine and Area Rules

Teaching Approach

The Sine, Cosine and Area Rules are covered in the third term of over a period of three weeks.

When teaching trigonometry to learners it is important that you give learners work in different contexts. Help learners translate the word problems into diagrams. If the diagrams are composite, for example in a quadrilateral, learners need to redraw triangles and label them appropriately. If possible, let learners do practical work on trigonometry.

There is a lot of information for learners and extra time should be taken to ensure that learners understand exactly how the concepts explained in these videos work in the real world.

The videos included in this series can be watched in any order. Summaries of the skills and contexts of each video are in this guide, allowing you to find something appropriate, quickly and easily. You will find a selection of tasks covering the required skills in the task video. These tasks have not been linked to the videos so that they can be used without viewing them.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:
- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. **Problems in Two Dimensions**

    This video helps learners correct the misconception of trying to apply trigonometric ratios to non-right-angled triangles. In this video, we discuss solving triangles using the trigonometric ratios and Pythagoras theorem.

2. **Working with the Sine Rule**

    This video proves and applies the Sine Rule for non-right angled triangles. Contextual questions have been given and learners are encouraged to sketch diagrams and label them.

3. **Working with the Cosine Rule**

    This video proves and applies the Cosine Rule for non-right angled triangles. Contextual questions are given so that understanding more than just the use of the formula.

4. **Working with the Area Rule**

    This video proves and applies the Area Rule for non-right angled triangles. Contextual problems are written so that learners can have conceptual understanding.

5. **Using the Sine, Cosine and Area Rules**

   This video shows the application of the three rules in contextualised problems.

Resource Material

|		
1. Problems in Two Dimensions	http://www.onlinemathlearning.com/basic-trigonometry.html	Comprehensive revision on all prior Trigonometry. Includes worksheets, notes and videos.
2. Working with the Sine Rule	http://www.cimt.plymouth.ac.uk/projects/mepres/step-up/sect4/index.htm	Shows step by step usage of Sine and Cosine Rules.
	http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/furthertrigonometryhirev1.shtml	Explanation of Sine Rule and questions using the rule.
	http://www.wikihow.com/Use-the-Sine-Rule	Step by step usage of the Sine Rule.
	http://www.mathsisfun.com/algebra/trig-sine-law.html	Step by step usage of the Sine Rule with questions.
3. Working with the Cosine Rule	http://www.cimt.plymouth.ac.uk/projects/mepres/step-up/sect4/index.htm	Shows step by step usage of Sine and Cosine Rules.
	http://www.mathstat.strath.ac.uk/basicmaths/332_sineandcosinerules.html	Shows step by step usage of Sine and Cosine Rules.
	http://www.mathsisfun.com/algebra/trig-cosine-law.html	Step by step usage of the Cosine Rule with questions.
	http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/furthertrigonometryhirev2.shtml	Explanation of the Cosine Rule.
4. Working with the Area Rule	http://www.regentsprep.org/Regents/math/algtrig/ATT13/areatriglesson.htm	An explanation on how to use the Area Rule.
	http://www.bbc.co.uk/bitesize/standard/maths_ii/trigonometry/sin_cosine_area_triangle/revision/3/	An example of how to use the Area Rule.
5. Using the Sine, Cosine and Area Rules	http://everythingmaths.co.za/grade-11/06-trigonometry/06-trigonometry-05.cnxmlplus	Summary of the three rules and questions that require the use of all three.
	http://www.banchoryacademy.co.uk/Int2%20Trigonometry%20worksheet.pdf	Questions involving all three rules.

Task

Question 1

In the diagram, PM is perpendicular to the horizontal plane LMN. If PM = 13metres, $P\widehat{N}M = 60^0$, $M\widehat{N}L = 34^0$ and $M\widehat{L}N = 40^0$.
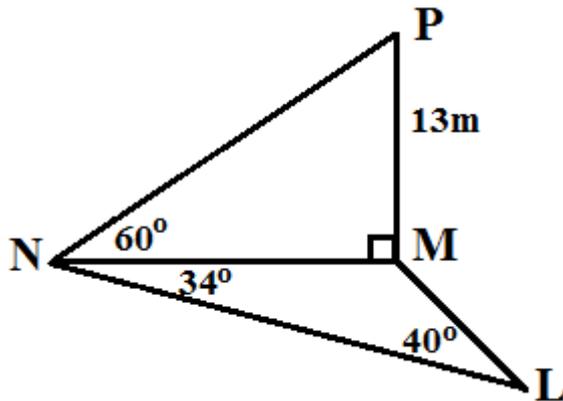
1.1 Find the length of LN
1.2 Find the area of ΔLMN
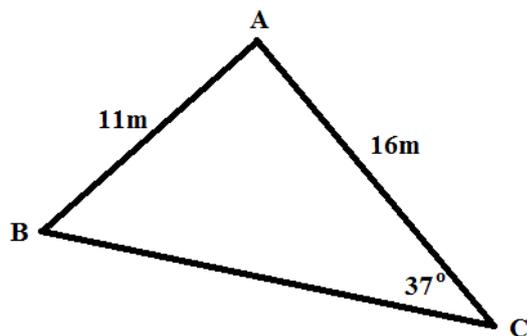
Question 2

In the diagram, a triangular garden ABC is given. AB = 11m; AC = 16m and $A\widehat{C}B = 37^0$
Find:
2.1 $B\widehat{A}C$
2.2 The distance BC
2.3 The area of triangle ABC

Question 3

An up market mall will be built on a piece of land that is in the shape of a parallelogram. The shorter measures 1,5 km long and the longer side is 2,5 km. One of the angles is 75^0. The planners decide to put a road through the shorter diagonal.
3.1 Find the area of the piece of land
3.2 Calculate the length of the road that goes through the piece of land.
3.3 Calculate the angles of each triangle that is formed when the road is built.

Question 4
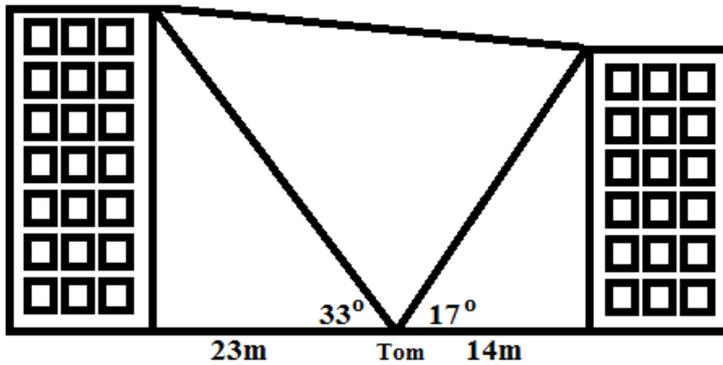
Tom stands between two buildings of different heights. He is 23m away from the building on his left and the angle of elevation between him and the building is 33°. The building on the right is 14m away and is at an angle of elevation is 17°. Find:

4.1 Find the heights of the two buildings

4.2 Calculate the distance from Tom to the top of the building on his left.

4.3 Calculate the length of a piece of rope which connects the tops of the two buildings.

Task Answers

Question 1

1.1 To calculate the length of LN we need the distance MN .
MN is the adjacent side to angle PNM and PM is the opposite side.
Therefore we should use the tangent ratio

$\tan\theta = \frac{opposite}{adjacent}$

$\tan 60^0 = \frac{13}{MN}$ $\therefore$ MN $= \frac{13}{tan60}$ = 7,5055m

we can either use the sine rule or the cosine rule to find the length of LN.

For the sine rule let us first find the $N\widehat{M}L$
$N\widehat{M}L = 180^0 - (34^0 + 40^0) = 106^0$

$$\frac{a}{SinA} = \frac{b}{SinB}$$

$$\frac{LN}{sin106^0} = \frac{7,5055}{sin40^0}$$

$$LN = \frac{7,5055 \times \sin 106}{sin40}$$
$$LN = 11,22m \quad (to \ 2 \ decimal \ places).$$

Or If we want to use the cosine rule we should start by finding the side LM
$$\frac{LM}{sinN} = \frac{MN}{sinL}$$
$$\frac{LM}{sin34} = \frac{7,5055}{sin40}$$
$$LM = \frac{7,5055 \times sin34}{sin40}$$
$$LM = 6,5294m$$

$$LN^2 = LM^2 + MN^2 - 2 \times LM \times MN \times \cos M$$
$$LN^2 = 7,5055^2 + 6,5294^2 - 2 \times 7,5055 \times 6,5294 \times cos106$$
$$LN^2 = 125,9815901$$
$$LN = 11,22m$$

So the answers we get are the same.

1.2 $Area = \frac{1}{2}absin\theta$

$$Area = \frac{1}{2} \times 7,5055 \times 11,22m \times sin34^0$$
$$Area = 23,55m^2$$

Question 2

2.1 We should calculate the angle ABC first since we have a and c.

$$\frac{SinB}{b} = \frac{sinC}{c}$$

$$\frac{sinB}{16} = \frac{sin37^0}{11}$$

$$sinB = \frac{16 \times sin37}{11}$$

$$A\hat{B}C = 61,09^0$$

$$\therefore B\hat{A}C = 180^0 - (61,09^0 + 37^0) = 81,91^0$$
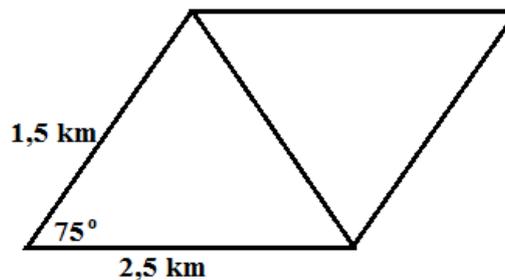
2.2 Distance BC, we use the cosine rule.

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times cosA$$

$$BC^2 = 16^2 + 11^2 - 2 \times 16 \times 11 \times cos81,91$$

$$BC^2 = 327,4635898$$

$$\therefore BC = 18,095m$$

2.3 Area of ABC $= \frac{1}{2} \times 16 \times 11 \times sin81,91$

Area of ABC $= 87,12m^2$

Question 3

3.1 A parallelogram is made up of two congruent triangles if cut on the diagonal. Since the area would be twice that of a triangle the formula should be $Area = absin\theta$.

$$Area\ of\ the\ parallelogram = 1,5 \times 2,5 \times sin75$$

$$Area\ of\ the\ parallelogram = 3,62km^2$$

3.2 The shorter diagonal would be the one that's opposite a smaller angle.

$$d^2 = 1,5^2 \times 2,5^2 - 2 \times 1,5 \times 2,5 \times cos75$$

$$d^2 = 6,558857162$$

$$\therefore d = 2,56km$$

3.3 The angles would be calculated using the sine rule.

$$\frac{sinA}{1,5} = \frac{sin75^0}{2,56}$$

$$sinA = \frac{1,5sin75^0}{2,56}$$

$$\hat{A} = 34,47^0$$

$$The\ other\ angle\ would\ be\ 180^0 - (75^0 + 34,47^0) = 70,53^0$$

Question 4

4.1 The height of the building on the right $= 14 \times \sin 17^0 = 4,09\text{m}$

    The height of the building on the let $= 23 \times \tan 33^0 = 14,94\text{m}$

4.2 The distance of Tom to the top of the building $= \frac{23}{\cos 33^0} = 27,42m$

4.3 The distance : $d^2 = 14^2 + 27,42^2 - 2 \times 14 \times 27,42 \times \cos 130^0$

                     $= 1441,363015$

               Distance $= 37,97\text{m}$

4.4 Area enclosed $= 0,5 \times 14 \times 27,42 \times \sin 130^0 = 147,03\text{m}^2$