A Guide to Circle Geometry

Teaching Approach

In Paper 2, Euclidean Geometry should comprise 35 marks of a total of 150 in Grade 11 and 40 out of 150 in Grade 12. This section of Mathematics requires both rote learning as well as continuous practice. Pen and paper repetition is the best way to get this right. Each pupil should have a correct handwritten copy of every theorem to refer to and to memorise. The theorems and their proofs, as well as the statement of the converses, must be learned for examination purposes. The theorems, converses, and other axioms must be used to solve riders and should also be used in formal proofs. In these cases the correct (and understandable) abbreviation of a theorem or its corollary can be used.

The emphasis during assessment will be on the correctness of formal arguments in examinations and notation will be scrutinised carefully. In most cases the student needs to follow the statement, reason, conclusion format. Please refer to the task answers to ensure correct setting out is followed. The numbering of theorems is author dependent and the reasons 'Theorem 3' will not be acceptable; neither will reasons like 'bow tie' or 'wind surfer'. All the results and definitions from previous grades are acceptable as axioms and do not need to be proved for the circle geometry results.

The proofs of the theorems should be introduced only after a number of numerical and literal riders have been completed and the learners are comfortable with the application of the theory. When attempting a rider, it is a good idea to use colour to denote angles which are equal as well as cyclic quads, tangents etc. This will assist the learner in making the visual connections. An alternate method is to let one angle be equal to a variable, for example x, and continue around the diagram until all required angles are known. In all cases teachers must ensure that sketches and diagrams are legible. The skill we are aiming toward is then for the learner to write up your findings in a formal manner.

"The more I practise, the luckier I get" is Gary Player's quote for golf, but very applicable to this section of the syllabus.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Introducing Circle Geometry

In this video we cover three topics: Firstly the origins and uses of Euclidian Geometry and more specifically circle geometry; secondly the concept of a formal proof and the importance thereof and lastly, the terminology relating to a circle.

2. Chords and Radii

This video introduces the theorem relating to chords and radii. It is followed up with some examples of where the theory is applied.

3. Angles at Centre

This video covers the theorem dealing with perpendicular bisector of the chord, the meaning of the word 'subtends' and the theorem dealing with the angle at the centre of the circle. It concludes by applying the theorems to some examples.

4. Chords Subtending Angles

In this video we introduce the theorem which considers the angles subtended by chords at the circumference and then apply it to some examples.

5. Interior Angles in Cyclic Quadrilaterals

In this video we introduce the idea of a cyclic quadrilateral; one theorem related to it and then we apply the theorem to some examples.

6. Exterior Angles in Cyclic Quadrilaterals

The theorem dealing with the opposite interior angles of a cyclic quadrilateral is discussed and some examples are worked through.

7. Proving Cyclic Quadrilaterals

In this video we look at different ways of proving a quadrilateral is a cyclic quadrilateral. That means proving that all four of the vertices of a quadrilateral lie on the circumference of a circle.

8. Tangents from a Point

The axiom that a tangent and a radius at the point of contact are always perpendicular is discussed and then this is used to prove that the tangents from the same point are equal in length.

9. The Tan-Chord Theorem

The tan-chord theorem is discussed in this lesson.

10. The Converse Tan-Chord Theorem

This video deals with the converse of the tan-Chord theorem and an examination style question is worked through.

11. Working with Circle Geometry

A problem which combines a number of bits of theory is dealt with and then the presenter reflects on how to approach problems in an examination.

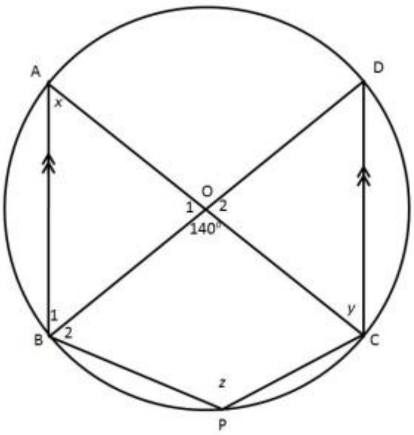
Resource Material

http://everythingmaths.co.za/grade-10/11-geometry/11-geometry-01.cnxmlplus	A summary of pre-Grade 10 geometry
http://math.about.com/od/formulas/ss/surfaceareavol.htmhttps://www.khanacademy.org/math/geometry/segments-and-angles/intro_euclid/v/euclid-as-the-father-of-geometry	Khan Academy gives video clips on all sections of geometry
http://www.youtube.com/watch?v=KvaD0x8QdDA	You tube presentation on chords and their perpendicular bisectors
http://www.youtube.com/watch?v=XRk5zhXzhWA	You tube presentation on inscribed and central angles
http://m.everythingmaths.co.za/grade-11/08-euclidean-geometry/08-euclidean-geometry-02.cnxmlplus	Text and video clips on all the terminology needed for this section, the formal proofs of theorems and worked examples.
http://maths911.com/paper3/circle%20geometry/Geometry%20Part%201.pdf	Summary of geometry done to this point and the first few theorems and examples
http://maths911.com/paper3/circle%20geometry/Geometry%20Part%202.pdf	Notes on angles subtended as well as examples
http://maths911.com/paper3/circle%20geometry/Circle%20Geometry%20Part%203.pdf	Notes on cyclic quads and examples

http://maths911.com/paper3/circle%20geometry/Circle%20Geometry%20Part%204.pdf	Notes on circle geometry - examples
http://www.youtube.com/watch?v=TiIq5i80JA4	You Tube presentation of the tan-chord theorem

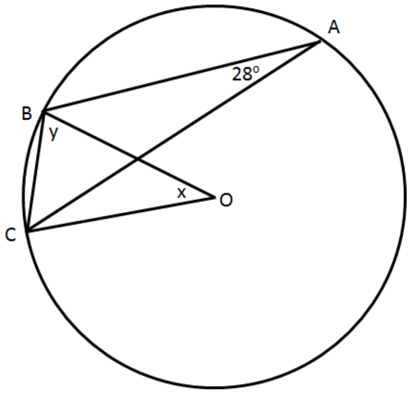
Task

Question 1



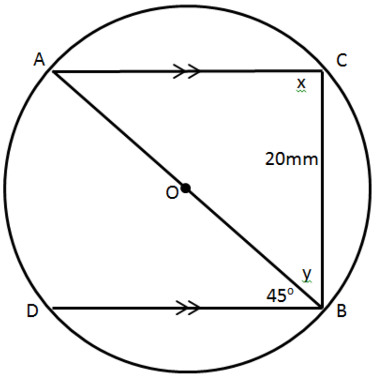
AC and BD are diameters. AB // CD and AC and meets BD at O. $B\hat{O}C = 140^o$. Find the numerical magnitude of the angles labelled x, y and z.

Question 2



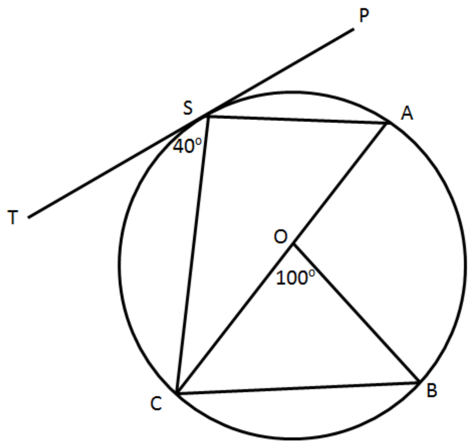
A, B and C are points on the circumference of the circle centre O. Find the numerical sizes of the angles labelled x and y.

Question 3



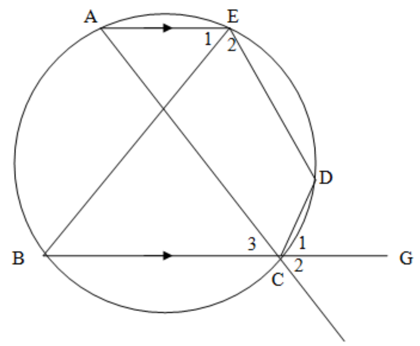
AOB is a diameter, BC = 20mm, AC // DB. Find the magnitudes of the angles labelled x and y and the lengths AC and OB.

Question 4



PT is a tangent to the circle at S. Prove that $SA \parallel CB$.

Question 5



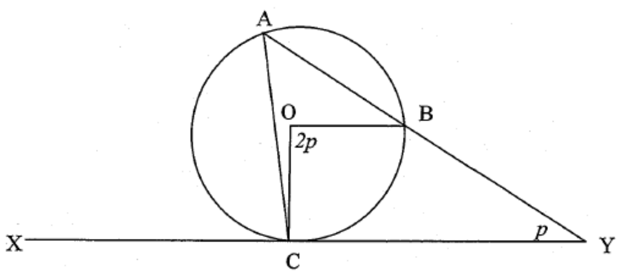
In the diagram $\hat{E}_1 = \hat{E}_2$ and $AE \parallel BC$. Prove:

5.1 $\hat{C}_1 = \hat{C}_2$

5.2 $BE \parallel DC$

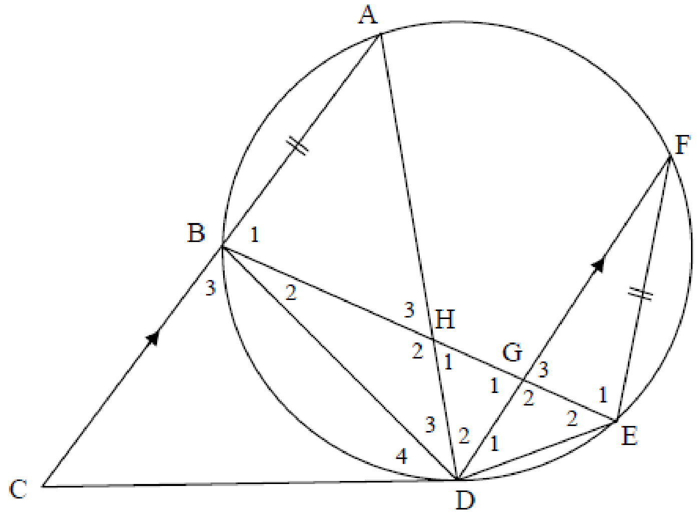
5.3 $E\hat{D}C = B\hat{C}D$

Question 6



XCY is a tangent to the circle centre O, $C\hat{O}B = 2p$ and $B\hat{Y}C = p$.Determine, in terms of p, the following three angles: $B\hat{A}C,\ X\hat{C}A$ and $A\hat{C}O$.

Question 7



CD is a tangent to circle $ABDEF$ at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. AC // FD and $FE = AB$. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$

7.1 Determine THREE other angles that are each equal to x.

7.2 Prove that $\Delta BHD\ ///\ \Delta FED$

7.3 Hence, or otherwise, prove that $AB \times BD = FD \times BH$

Task Answers:

Question 1

$x = 70^o$ L's at centre 2X L's at circum.

$y = 70^o$ Alt L's AB//DC

$z = \frac{220^o}{2} = 110^o$ L's at centre, reflex

Question 2

$x = 56^o$ L's at centre 2X L's at circum

ΔBOC is isos Radii OB=OC

$\therefore y = 62^o$ L's in Δ

Question 3

$x = 90^o$ Diam subtends R L's

$\hat{A} = 45^o$ Alt L's AC//DB

$\therefore y = 45^o$ L's in Δ

AC = 20mm Isos Δ

$AB = \sqrt{20^2 + 20^2} = \sqrt{800}$ Pythag

$\therefore OB = 56.57$mm

Question 4

$\hat{A} = 40^o$ Tan chord

ΔOCB Isos Radii OC = OB

$\therefore \hat{C} = 40^o$

$\therefore$ SA//CB Alt L's equal

Question 5

5.1 $\hat{B} = \hat{E}_1$ Alt L's = AE//BC

$\hat{E}_2 = \hat{C}_1$ Ext L's cyclic quad

$\hat{E}_1 + \hat{E}_2 = \hat{C}_1 + \hat{C}_2$ Ext L's cyclic quad

But $\hat{E}_1 = \hat{E}_2$

$\therefore \hat{C}_1 = \hat{C}_2$

5.2 $\hat{B} = \hat{E}_1$ Alt L's = AE//BC

$\hat{C}_1 = \hat{E}_1$ proved above

$\therefore \hat{B} = \hat{C}_1$

$\therefore$BE//DC Corresp L's =

5.3 $\hat{B} + \hat{D} = 180^o$ oppo ∠'s cyclic quad sup

$B\hat{C}D + \hat{C}_1 = 180^o$ co-int ∠'s sup

and $\hat{B} = \hat{C}_1$ above

$\therefore B\hat{C}D = \hat{D}$

Question 6

$B\hat{A}C = p$ ∠'s at centre

$Y\hat{C}A = 2p$ ext ∠'s Δ

$A\hat{C}O = 90^o - 2p$ tangent ⊥ radius

Question 7

7.1 $\hat{A} = \hat{D}_4 = x$ tan chord theorem

$\hat{E}_2 = x$ tan chord theorem or ∠'s in same segment

$\hat{D}_2 = \hat{A} = x$ Alt ∠'s, CA//DF

7.2 In ΔBHD and ΔFED

$\hat{B}_2 = \hat{F}$ ∠'s in same segment

$\hat{D}_3 = \hat{D}_1$ =chds subt = ∠'s

$\therefore \Delta BHD /// \Delta FED$ ∠∠∠

7.3 $\frac{FE}{BH} = \frac{FD}{BD}$ ///Δ's

but $FE = AB$ given

$\therefore \frac{AB}{BH} = \frac{FD}{BD}$

AB.BD = FD.BH

Acknowledgements

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