

A Guide to Advanced Algebraic Functions

The section, functions, is an incredibly important part of the CAPS curriculum. It should not be taught in isolation but rather linked to the algebraic concepts already taught. Learners should be taught how quadratic equations, factorising and transformations form part of this section. During teaching it is a good idea to emphasize these links.

The section, Advanced Algebraic Functions, is divided into five series, Revising Algebraic Functions, The Quadratic Function, The Hyperbolic Function, The Exponential Function and Mixed Algebraic Functions.

The series begins by reviewing the basic knowledge that learners should know from previous grades. The first lesson covers in depth all the knowledge that learners should have learnt in Grade 10. This includes a detailed summary that we recommend learner to copy and use as a reference. We then start with the Grade 11 content. We begin by investigating the quadratic function, its standard equations, how to sketch these graphs and how to determine the equation of this function. We follow with the hyperbola and exponential functions. Once learners are familiar with the fundamental concepts of these three functions we teach learners how to use what they know about the effects of the variables, a, p and q to interpret graphs.

These functions may be familiar to some of your learners. However, they may have learnt about the properties of the functions without really investigating for themselves. This section provides many opportunities to challenge your learners' thinking. In the video lessons, we use a picture of a pause button where we think you might want to stop the tape to have a class discussion, or ask learners to complete an activity or to copy down notes and theory.

It is also important to note that standard forms of the equations have changed, CAPS requires us to teach learners the following equations:

$$y = ax^2 + bx + c$$

$$y = a(x + p)^2 + q$$

$$y = \frac{a}{x + p} + q$$

$$y = a \cdot b^{x+p} + q$$

Many of the new resources that are CAPS compliant have still included the old standard equations. It is important to show learners the differences so that they are not confused when using older resources such as study guides or past papers. It is also important to ensure that learners use and understand the different notations such as mapping and functional notation.

To get the full benefit of the lessons, your learners need to engage actively with the concepts presented. So, when you preview the videos, think about how to introduce each lesson and what follow up activities will be useful.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

Recapping Algebraic Functions

1. Revising Grade 10 Functions

The lesson explores the standard equations of the quadratic, hyperbolic and exponential functions. Summaries in a table form are given discussing the effects of each variable in the equation as well domain and range.

The Quadratic Function

1. Revision of Solving Quadratic Equations

In this lesson we investigate the standard and the turning point forms of the quadratic formula. We then look at the properties of these formulae and the graph by doing examples.

2. Revising Completing the Square

In this lesson learners are taught about completing the square. We complete the square to change between the equation in the.

3. Sketching the Quadratic Function

In this lesson learners are shown how to sketch the quadratic function in four simple steps. Examples are covered in detail and worked through step by step.

4. Determining the Equation of a Quadratic Function I

This lesson shows learners how to determine the equation of a quadratic function if the coordinates of the turning point and another coordinate are given.

5. Determining the Equation of a Quadratic Function II

This lesson covers two examples where learners are taught how to find the equation of a parabola when given the x – intercept and another point.

6. Determining the Equation of a Quadratic Function III

This lesson covers two examples where learners are taught how to find the equation of a parabola when given the y – intercept and two other points.

The Hyperbolic Function

1. Investigating the Hyperbolic Function

This lesson helps learners understand that the hyperbola can be shifted left and right as well as up and down. We investigate the standard form of the equation and the properties of this graph.

2. Summarising the Hyperbolic Function

The lesson includes a table summary of all the properties of the hyperbola and ends off with a few questions for learners to apply what they have learnt in the lesson.

3. Sketching the Hyperbolic Function

This lesson shows how to sketch the hyperbola in six simple steps. Examples are covered in detail and worked through step by step.

4. Determining the Equation of a Hyperbolic Function

This lesson covers two examples where learners are taught how to find the equation of a hyperbola.

The Exponential Function

1. Investigating the Exponential Function

This lesson shows that the exponential graph can be shifted left and right as well as up and down. The lesson includes a table summary of all the properties of the exponential graph and ends off with a few questions for learners to apply what they have learnt in the lesson.

2. Sketching the Exponential Function

Learners are shown how to sketch the exponential graph in six simple steps. Examples are covered in detail and worked through step by step.

3. Determining the Equation of an Exponential Function

This lesson covers two examples where learners are taught how to find the equation of an exponential function.

Mixed Algebraic Functions

1. Interpreting Mixed Graphs I

In this lesson learners have the opportunity to apply combinations of vertical and horizontal translations as well as reflections of quadratic functions. Interpret formulae of the relevant type to describe the functions.

2. Interpreting Mixed Graphs II

This lesson we examines a set of axes with a parabola, straight line and a circle, plotted on them.

3. The Average Gradient between Two Points

This lesson explores the average gradient of a straight line between two points on a graph and the concept of average rate of change.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

Recapping Algebraic Functions

1. Revising Grade 10 Functions	http://www.mathsisfun.com/sets/function.html	This website has various examples to help learners understand by the word function.
	http://www.youtube.com/watch?v=fFJ3xZXQ8qY	YouTube video explaining functions, input and output with an example
	http://www.bymath.com/studyguide/fun/sec/fun9.htm	A resource for teachers who want to brush up on their personal knowledge regarding functions .

The Quadratic Function

1. Revision of Solving Quadratic Equations	http://mrpilarski.wordpress.com/2010/01/05/using-properties-of-parabolas-to-graph-a-parabola/	Summary of the properties of the parabola.
2. Revising Completing the Square	http://www.algebra.com/algebra/homework/quadratic/Quadratic_Equations.faq.question.1829.html	This is a good resource to use for examples on finding the equation of a graph.
3. Sketching the Quadratic Function	http://www.youtube.com/watch?v=8HvZHo-LSvQ	A YouTube video showing how to draw parabola, not a resource to be used with learners as the different American terminology may confuse them.
	http://www.purplemath.com/modules/grphquad.htm	How to draw a parabola using their basic knowledge.

4. Determining the Equation of a Quadratic Function I	http://analyzemath.com/parabola/FindEqParabola.html	A good resource, as one can generate multiple questions on an applet for finding the equation of parabola graphs.
5. Determining the Equation of a Quadratic Function II		
6. Determining the Equation of a Quadratic Function III		

The Hyperbolic Function

1 Investigating the Hyperbolic Function	https://everythingmaths.co.za/grade-11/05-functions/05-functions-04.cnxm plus	Everything Maths chapter focusing on hyperbolas.
2 Summarising the Hyperbolic Function	https://www.youtube.com/watch?v=04CaUOonMOc&list=PLOaNAktW5HLSnwMEtKTDaZBq0zzIJIPHx	Mindset Learn Xtra lesson on functions.
3 Sketching the Hyperbolic Function	https://www.youtube.com/watch?v=HioQMb1as4U&list=PLOaNAktW5HLSv0_5mAwrVPFIWkt2J3KC&index=8	Mindset Learn lesson on the Hyperbola and other functions.
4 Determining the Equation of the Hyperbolic Function	http://www.mathsexcellence.co.za/papers/graphs/GR11_GRAPHs1.pdf	PDF notes on the parabola and hyperbola.

The Exponential Function

1. Investigating the Exponential Function	http://www.purplemath.com/modules/expofcns.htm	A resource that explores the different shapes of the exponential graph by investigating the variables in the equation.
	http://www.mathworks.com/moler/exm/chapters/exponential.pdf	A detailed look at the exponential function and examples in real world context. A nice resource for educators to use to improve their knowledge of this function.
	http://www.mathsisfun.com/sets/function-exponential.html	A resource that is in a simple format that can be used with the learners for them to further understand the exponential function.
2. Sketching the Exponential Function	http://www.analyzemath.com/Graphing/GraphExponentialFunction.html	A resource that provides a step by step tutorial on sketching the exponential graph. The properties such as domain, range, horizontal asymptotes and intercepts of the graphs of these functions are also examined in details.

3. Determining the Equation of an Exponential Function	http://wcherry.math.unt.edu/math1650/exponential.pdf	A resource to be used by teachers to brush up on their knowledge, a little complicated for learners.
	http://www.analyzemath.com/expfunction/find_exponential_function.html	Find the equation of exponential graphs
<i>Mixed Algebraic Functions</i>		
1. Interpreting Mixed Graphs I	http://www.purplemath.com/modules/fcntrans.htm	Various transformation are explored and investigated
	http://www.padowan.dk/	Free software that can be downloaded which draws graphs. Very easy to use.
	http://www.mathsisfun.com/sets/function-transformations.html	A colourful resource that shows clearly the transformation by colour coding.
2. Interpreting Mixed Graphs II	http://www.greatmathsteachingideas.com/wp-content/uploads/2012/03/A7.pdf	This resource provides ideas for teachers that can be used in the classroom.
3. The Average Gradient between Two Points	http://www.everythingmaths.co.za/grade-11/14-gradient-at-a-point/14-gradient-at-a-point-02.cnxmplus	The South African text book, Everything Maths.
	http://www.mathsisfun.com/gradient.html	This resource can be used to revise the concept of gradient before moving onto gradient between two points.

Task
Question 1

Sketch the following functions, show and label all intercepts and the turning point.

1.1 $f(x) = \frac{1}{2}(x + 3)^2 - 2$

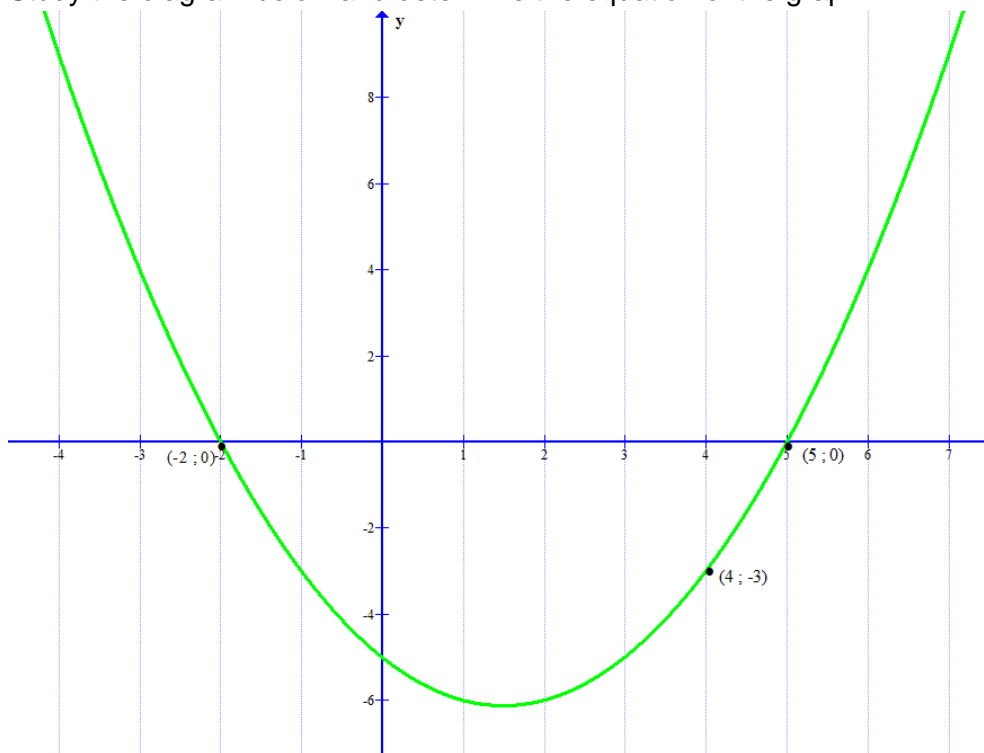
1.2 $g(x) = -2x^2 + 8x + 20$

Question 2

Determine the equation of the quadratic function $h(x)$ if the turning point is $(-3 ; 2)$ and the point $(-2 ; 4)$ lies on the graph h . Give your answer in the form of $h(x) = ax^2 + bx + c$

Question 3

Study the diagram below and determine the equation of the graph.


Question 4

Draw a sketch of:

4.1 $f: x \rightarrow \frac{6}{x-1} - 3$. Show and label all intercepts with the axes and asymptotes.

4.2 $p(x) = -(2)^{x+1} - 2$. Show and label all intercepts with the axes and asymptotes.

Question 5

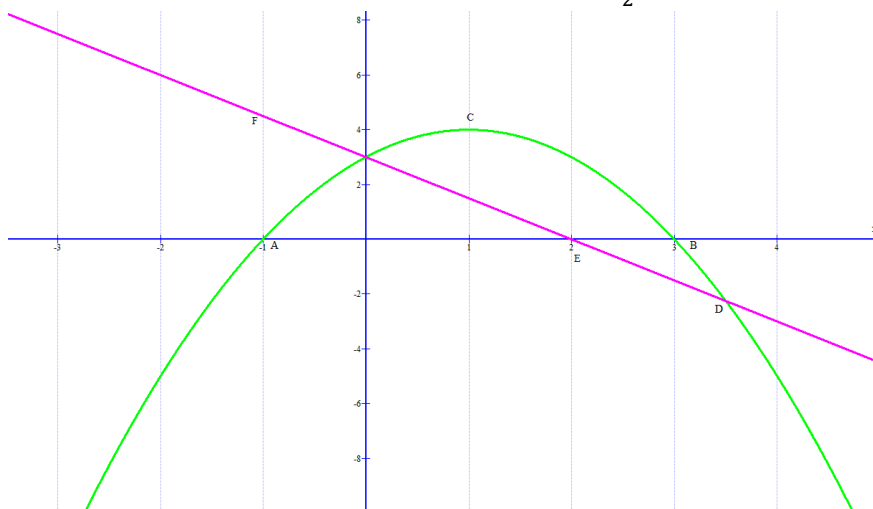
Determine the equation of $g(x) = \frac{a}{x+p} + q$ if the asymptotes are $y = -6$ and $x = 4$. And point $A(5; -4)$ lies on $g(x)$.

Question 6

Draw a sketch of $f(x) = ax^2 + bx + c$ if $a < 0$; $b < 0$ and $c > 0$

Question 7

The graph of $f: x \rightarrow -x^2 + 2x + 3$ and $g: x \rightarrow -\frac{3}{2}x + 3$ are shown below.

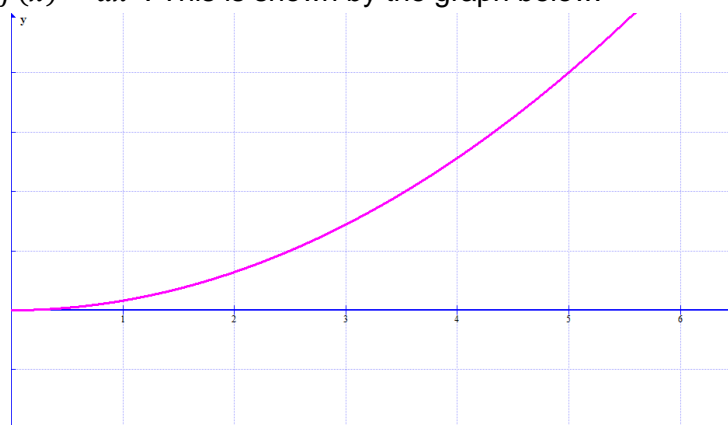


Calculate:

- 7.1 The coordinates of A, B and C.
- 7.2 The length of FA
- 7.3 The coordinates of D and E
- 7.4 The value of x for which $(f: x)(g: x) \leq 0$

Question 8

The take off distance of a golf ball, when hit by the average amateur, is given by $f(x) = ax^2$. This is shown by the graph below.

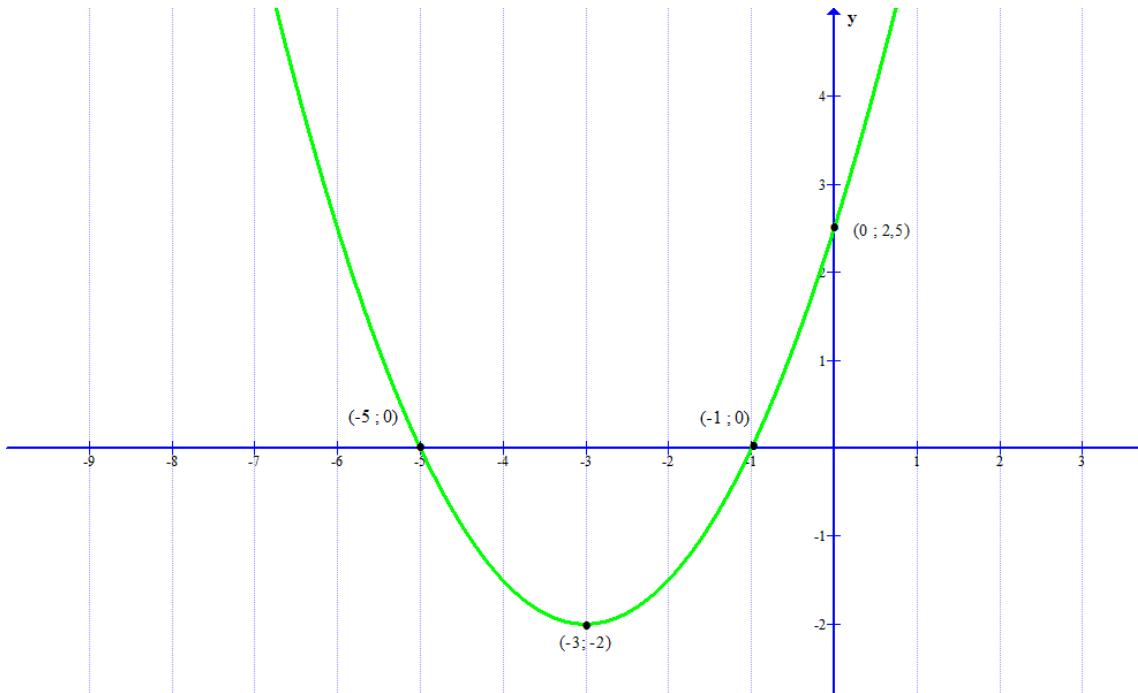


- 8.1 Determine the value of a
- 8.2 What distance has the golf ball travelled after 3sec?
- 8.3 What is the average gradient of the ball during take off
- 8.4 What does the answer in question 8.3 represent?

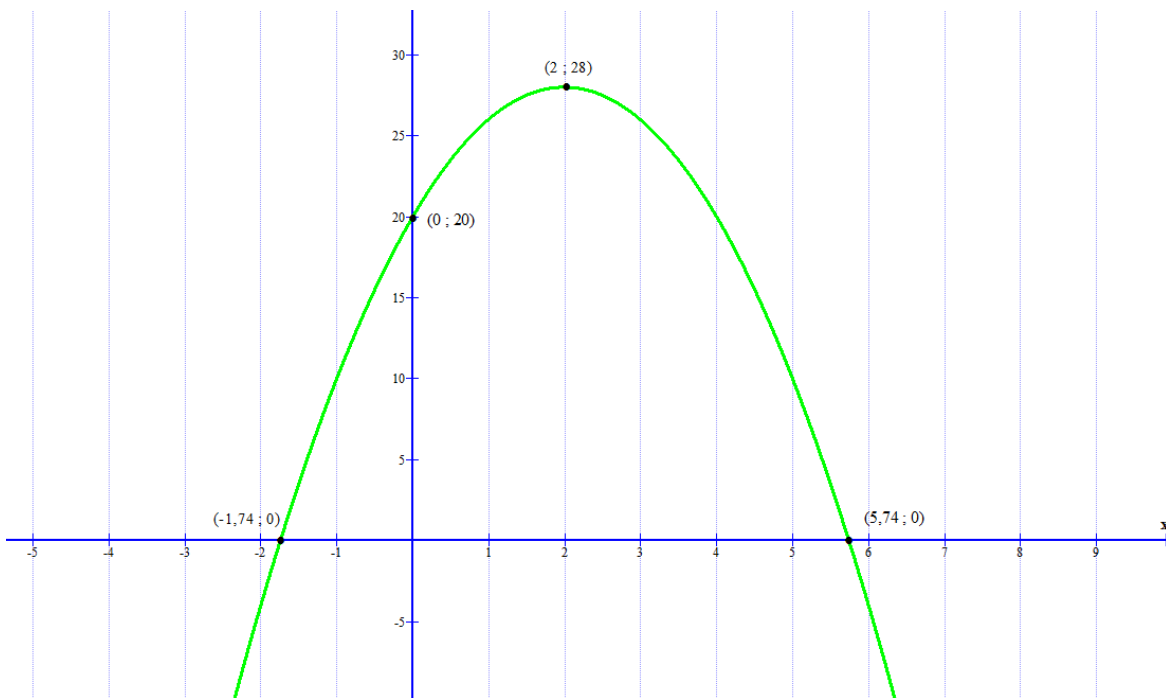
Task Answers

Question 1

1.1 .



1.2



Question 2

$$y = a(x + p)^2 + q$$

$$y = a(x + 3)^2 + 2$$

$$4 = a(-2 + 3)^2 + 2$$

$$4 = 1a + 2$$

$$2 = a$$

$$y = 2(x + 3)^2 + 2$$

$$\therefore y = 2x^2 + 12x + 20$$

Question 3

$$y = a(x - r_1)(x - r_2)$$

$$y = a(x - (-2))(x - (5))$$

$$-3 = a(4 + 2)(4 - 5)$$

$$-3 = a(6)(-1)$$

$$-3 = -6a$$

$$\frac{1}{2} = a$$

$$\therefore y = \frac{1}{2}(x + 2)(x - 5)$$

Question 4

4.1. $x = 1$

$$y = -3$$

$$0 = \frac{6}{x-1} - 3$$

$$3 = \frac{6}{x-1}$$

$$3(x - 1) = 6$$

$$3x - 3 = 6$$

$$3x = 9$$

$$x = 3$$

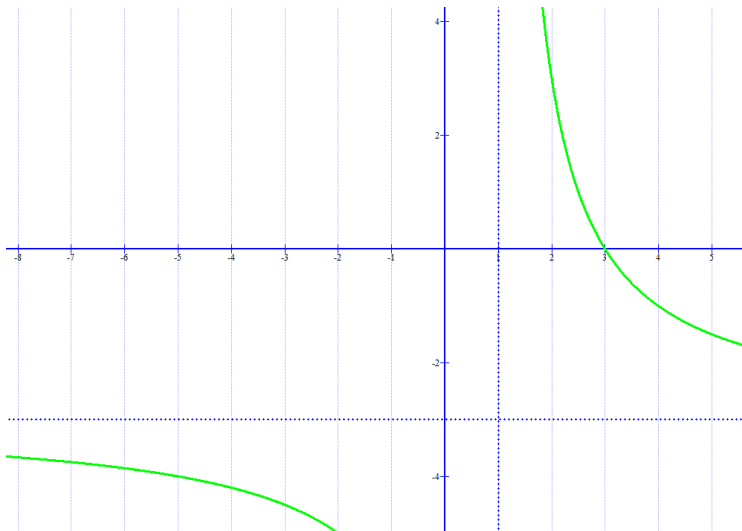
(3 ; 0)

$$y = \frac{6}{0-1} - 3$$

$$y = -6 - 3$$

$$y = -9$$

(0 ; -9)



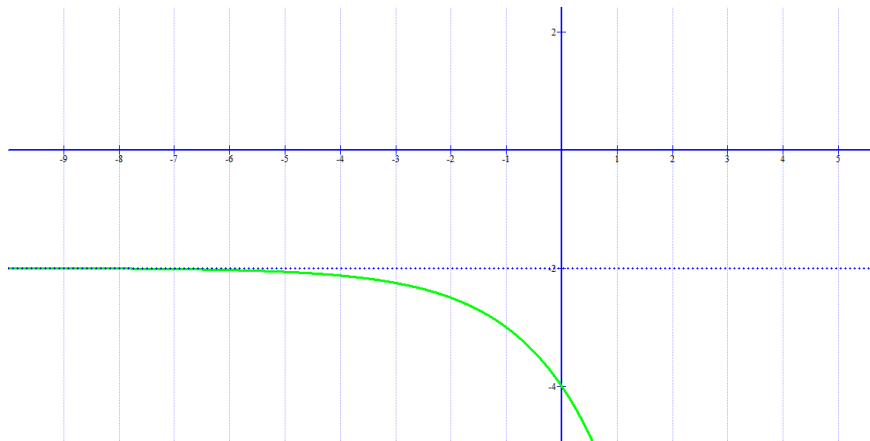
4.2. $y = -2$

$$y = -(2)^{0+1} - 2$$

$$y = -2 - 2$$

$$y = -4$$

(0 ; -4)



Question 5

$$y = \frac{a}{x+(-4)} - 6$$

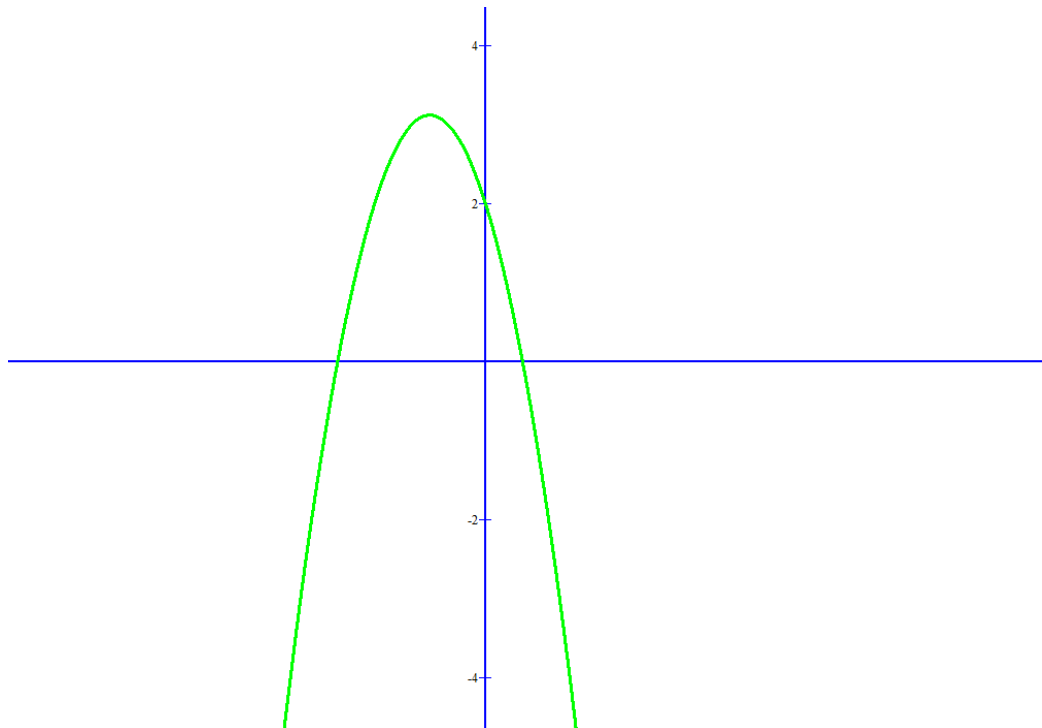
$$-4 = \frac{a}{-4-4} - 6$$

$$2 = \frac{a}{-8}$$

$$-16 = a$$

$$\therefore y = \frac{-16}{x-4} - 6$$

Question 6



Question 7

7.1. $f(x) = -x^2 + 2x + 3$

$$-x^2 + 2x + 3 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$\therefore x = 3 \text{ or } x = -1$$

A (-1 ; 0)

B (3; 0)

$$x = -\frac{b}{2a}$$

$$x = -\frac{2}{2(-1)}$$

$$x = 1$$

$$f(1) = -(1)^2 + 2(1) + 3$$

$$= 4$$

TP C (1; 4)

$$7.2. g(x) - f(x) = FA$$

$$-\frac{3}{2}x + 3 - (-x^2 + 2x + 3) = FA$$

$$-\frac{3}{2}x + 3 + x^2 - 2x - 3 = FA$$

$$x^2 - \frac{7}{2}x = FA$$

$$(-1)^2 - \frac{7}{2}(-1) = FA$$

$$4\frac{1}{2} = FA$$

$$7.3. g(x) = -\frac{3}{2}x + 3$$

$$-\frac{3}{2}x + 3 = 0$$

$$-\frac{3}{2}x = -3$$

$$x = 2$$

$$\therefore E(2; 0)$$

$$g(x) = f(x)$$

$$-\frac{3}{2}x + 3 = -x^2 + 2x + 3$$

$$-\frac{3}{2}x + 3 + x^2 - 2x - 3 = 0$$

$$x^2 - \frac{7}{2}x = 0$$

$$x\left(x - \frac{7}{2}\right) = 0$$

$$\therefore x \neq 0 \text{ or } x = 3\frac{1}{2}$$

$$D\left(3\frac{1}{2}; -\frac{9}{4}\right)$$

$$7.4. x \leq -1 \text{ or } 2 \leq x \leq 3$$

Question 8

$$8.1. f(5) = a(5)^2$$

$$4 = 25a$$

$$a = \frac{4}{25}$$

$$8.2. f(3) = \frac{4}{25}(3)$$

$$= \frac{12}{25}$$

$$= 0,48\text{m}$$

$$8.3. m = \frac{5-0}{5,5-0}$$

$$= 0,9$$

8.4. The answer in 8.3, is the speed.

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