A Guide to Equations and Inequalities

Teaching Approach

When teaching the section of equations and inequalities, it is important to emphasise that we are solving for an unknown variable, and in a quadratic equation, we find two values for an unknown variable.

Completing the square

Completing the square is one method of solving for x in a quadratic equation. Usually, we only use this method when we are instructed to do so. The method of completing the square is a step by step process which the learners need to learn and must become familiar in applying through many practice examples. It is important to teach this as a step by step method and the learners must be very familiar with these steps.

The steps of completing the square involve:

- Getting the equation into the form where all the terms with the variable are on the left hand side of the equal sign, and any constant term (terms without a variable) are on the right.
- If there is a coefficient in front of the x^2 term, it must be taken out and all terms in the equation must be divided by this coefficient.
- The coefficient of the x term must now be multiplied by a half and squared, and then this answer must be added to the left and right hand side of the equation.
- The left hand side of the equation can now be factorised as a perfect square
- Finish off by solving for x.

Another reason for completing the square is to convert and quadratic equation, in the form $y = ax^2 + bx + c$, into the form of $y = a(x - p)^2 + q$. This equation is useful when identifying the coordinates of the turning point of a parabola.

The steps of completing the square to convert from $y = ax^2 + bx + c$ to $y = a(x - p)^2 + q$

- If there is a co-efficient of x, take it out but make sure that it is not dropped as the equation is not equal to zero
- The coefficient of the x term must now be multiplied by a half and squared, and then this answer must be both added and subtracted to the same side of the equation.
- Group and factorise the first three terms of the expression to make a perfect square
- Simplify the remaining terms.
- The equation should now be in the form of $y = a(x - p)^2 + q$.

Solving quadratic equations by factorising and the quadratic formula

Revise the methods of factorising namely, HCF, difference of two squares, trinomials, grouping and the sum or difference of two cubes.

Once the methods of factorising are revised introduce learners to the method of solving quadratic equations by factorising.

Teach learners to ensure that, firstly, the equation must be in the standard form of

$$ax^2 + bx + c = 0$$

Next, the equation must be factorised and then each bracket is made equal to 0 to create two linear equations. Each linear equation is then solved.

If an equation cannot be solved by factorising, the learners must then use the quadratic formula. The values for a, b and c from the equation must be substituted into the formula and the values of x are then either left in simplest surd form or they are rounded off to two decimal places, depending on the instruction given.

Inequalities

It must be emphasised that where an equation solves for specific values of x, and inequality solves for a range of possible values for x between certain points on the number line, which are called critical values. A quadratic inequality can be solved by a table of signs or graphically and the answer must be represented on a number line and/or in interval notation. It is recommended that the learners choose a method of solving a quadratic inequality that best suits their understanding and stick to it. This will avoid confusion between methods

Simultaneous equations

Simultaneous equations are necessary when there are two equations (usually one linear and one quadratic) which both have two different variables that must be solved for. The process of solving a quadratic equation involves isolating one of the variables in the linear equation and then substituting the value of that variable into the quadratic equation. Essentially, this process eliminates one of the variables, allowing us to solve for the one variable and then finally we can substitute the values we find back into either the linear or quadratic equation to solve for the other variable.

It is important to emphasise that we use simultaneous equations to solve for the coordinates of point(s) of intersection between two graphs.

Nature of roots

The learners need to understand that a 'root' is an answer for x in an equation. The 'nature of roots' therefore involves determining the **type** of number that the root is. We determine the nature of roots through the use of the discriminant, which is $b^2 - 4ac$, and it is usually denoted by the symbol Δ (Delta).

If $\Delta > 0$ the roots are real

If $\Delta > 0$ and is a perfect square, the roots are real and rational

If $\Delta < 0$ the roots are non-real

If $\Delta = 0$ the roots are equal

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Completing the Square

The concept of completing the square is introduced as a method to solve for the unknown variable in a quadratic equation. Examples are used to illustrate the step by step method of completing the square.

2. Revision of Solving Quadratic Equations

The concept of solving a quadratic equation through the use of factorisation is explained. The four methods of factorisation are revised and how to solve for an unknown variable once the quadratic equation is factorised.

3. The Quadratic Formula

This lesson looks at solving quadratic equations through the use of the quadratic formula.

4. Solving Quadratic Inequalities

The concept of quadratic inequalities is introduced and examples are done to illustrate the method/s of solving quadratic inequalities.

5. Solving Simultaneous Equations

Simultaneous equations are introduced and examples are done to show how two different variables are solved for simultaneously in a linear and a quadratic equation.

6. The Nature of Roots

In this video the concept of nature of roots are introduced. The learners are introduced to the discriminant, where it comes from and how it is used to determine the nature of the roots of a quadratic equation.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1 Completing the Square	http://www.mathwarehouse.com/quadratic/completing-the-square-math.php	Explanation and practice examples on the steps of completing the square
	http://www.mathsisfun.com/algebra/completing-square.html	Explanation on the steps of completing the square
	http://www.youtube.com/watch?v=Q7Sc7IX4TEk	Video on the method of completing the square
2 Revision of Solving Quadratic Equations	http://everythingmaths.co.za/grade-11/07-solving-quadratic-equations/07-solving-quadratic-equations-02.cnxmlplus	Explanation, examples and solutions of solving quadratic equations by factorization and the quadratic formula
	http://www.intmath.com/quadratic-equations/1-solving-quadratic-equations-factoring.php	Explanation and examples of solving for unknown variable by factorization and using the quadratic formula
3 The Quadratic Inequalities	http://www.intmath.com/quadratic-equations/3-quadratic-formula.php	Examples of solving for unknown variable by using the quadratic formula
	http://www.mathwarehouse.com/quadratic/solve-quadratic-equation-by-factoring.php	Video tutorial on solving a quadratic equation by factoring
4 Solving Quadratic Inequalities	http://www.youtube.com/watch?v=t54ccHYVhoo	Video tutorial on solving quadratic inequalities
	http://everythingmaths.co.za/grade-11/08-solving-quadratic-inequalities/08-solving-quadratic-inequalities-01.cnxmlplus	Explanation, examples and solutions on solving quadratic inequalities
	http://www.mathsisfun.com/algebra/inequality-quadratic-solving.html	Explanation on the various methods of solving a quadratic inequality
5 Solving Simultaneous Equations	http://everythingmaths.co.za/grade-11/09-solving-simultaneous-equations/09-solving-simultaneous-equations-01.cnxmlplus	Explanation, examples and solutions on solving simultaneous equations
	http://www.youtube.com/watch?v=8ockWpx2KKI	Video tutorial on solving simultaneous equations
	http://www.mathsrevision.net/gcse/pages.php?page=3	Explanation and examples of solving simultaneous equations
6 The Nature of Roots	http://www.tutorsonnet.com/math_homework_help/quadratic_equations/nature_of_roots_quadratic_equation_assignment_help_tutoring.htm	Explanation, examples and solutions of nature of roots
	http://www.youtube.com/watch?v=KKjS5x08Yv4	Video tutorial on determining the nature of roots
	http://lmstuition.weebly.com/nature-of-roots-and-discriminant.html	Explanation and examples on determining the nature of roots

Task

Question 1

Solve the following equation by completing the square. Leave your answer to two decimal places if necessary.

$x^2 - 3x = -5$

Question 2

Solve the following quadratic equation by completing the square. Leave your answer in simplest surd form.

$ax^2 + bx + c = 0$

Question 3

Solve for x:

$x(x-1) = 4(3x-10)$

Question 4

Solve the following equation. Leave your answer to two decimal places if necessary.

$-3p^2 + 7p - 4 = 0$

Question 5

Solve for a in the following equation. Consider all restrictions on the variables.

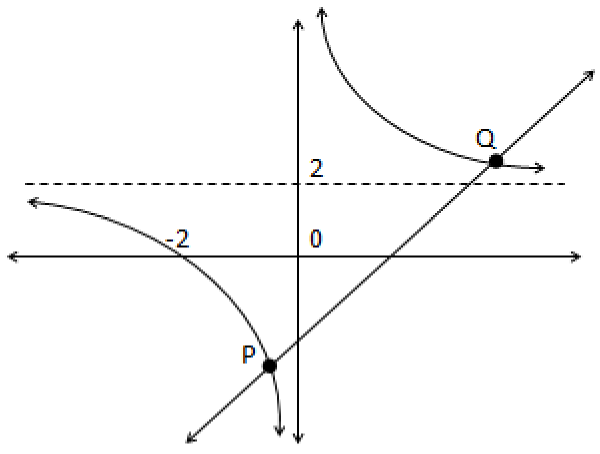
$$\frac{a}{a-2} = \frac{1}{a-3} - \frac{2}{2-a}$$

Question 6

Solve for x and y:

$x - 2y = -7$ and $x^2 + xy + y^2 = 21$

Question 7



In the above diagram, the functions of $y = \frac{4}{x} + 2$ and $y = x - 1$ are represented. Points P and Q are the points of intersection of the two graphs.

Find the coordinates of P and Q by solving the equations simultaneously.

Question 8

Solve the following inequalities. Represent your solution in interval notation and on a number line.

$x^3 + x^2 \geq 6x$

Question 9

Without solving for x, determine the nature of the roots of the following equations

9.1 $2x^2 + 5x + 10 = 0$

9.2 $x^2 - x = 6$

9.3 $4x^2 - 4x + 1 = 0$

Question 10

Prove that the roots of the equation$x^2 + k = (k + 2)x$ are real and unequal for all values of k.

Task Answers

Question 1

$x^2 - 3x = -5$
$x^2 - 3x + 9 = -5 + 9$
$(x-3)(x-3) = 4$
$(x-3)^2 = 4$
$x - 3 = \pm\sqrt{4}$
$x = \pm 2 + 3$
$\therefore x = 5 \; or \; x = 1$

Question 2

$ax^2 + bx + c = 0$
$a\left(x^2 + \frac{bx}{a} + \frac{c}{a}\right) = 0$
$x^2 + \frac{bx}{a} = \frac{-c}{a}$
$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$
$(x + \frac{b}{2a})^2 = \frac{-4ac + b^2}{4a^2}$
$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Question 3

$x(x-1) = 4(3x - 10)$
$x^2 - 1x = 12x - 40$
$x^2 - 13x + 40 = 0$
$(x-5)(x-8) = 0$
$\therefore \; x = 5 \; or \; x = 8$

Question 4

$-3p^2 + 7p - 4 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(-3)(-4)}}{2(-3)}$
$x = \frac{-7 \pm \sqrt{49 - 48}}{-6}$
$x = \frac{-7 \pm \sqrt{1}}{-6}$
$\therefore \; x = \frac{-6}{-6} \quad or \quad x = \frac{-8}{-6}$
$\therefore \; x = 1 \quad or \quad x = 1{,}33$

Question 5

$\frac{a}{a-2} = \frac{1}{a-3} - \frac{2}{2-a}$

$\frac{a}{a-2} = \frac{1}{a-3} + \frac{2}{a-2}$ $\quad$ $Restrictions: a \neq 2 \ or \ a \neq 3$

$\frac{a(a-3)}{(a-2)(a-3)} = \frac{1(a-2)+2(a-3)}{(a-2)(a-3)}$

$a^2 - 3a = a - 2 + 2a - 6$

$a^2 - 6a + 8 = 0$

$(a-2)(a-4) = 0$

$\therefore \ a \neq 2 \quad or \quad a = 4$

Question 6

$x - 2y = -7$ and $x^2 + xy + y^2 = 21$

$x = 2y - 7$ ... (1)

$x^2 + xy + y^2 = 21$.... (2)

Substitue equation 1 into equation 2

$\therefore (2y-7)^2 + (2y-7)y + y^2 = 21$

$\therefore \ 4y^2 - 28y + 49 + 2y^2 - 7y + y^2 = 21$

$\therefore \ 7y^2 - 35y + 28 = 0$

$\therefore \ y^2 - 5y + 4 = 0$

$\therefore \ (y-4)(y-1) = 0$

$\therefore \ y = 4 \ \ or \ \ y = 1$

Substitute $y = 4 \ \ or \ \ y = 1$ into equation 1

$\therefore \ x = 2(4) - 7$ or $x = 2(1) - 7$

$\therefore \ x = 1 \quad x = -5$

Question 7

$y = \frac{4}{x} + 2$.... (1)

$y = x - 1$... (2)

Substitute equation 1 into equation 2

$\therefore \frac{4}{x} + 2 = x - 1$

$\therefore 4 + 2x = x^2 - 1x$

$\therefore \ 0 = x^2 - 3x - 4$

$\therefore \ 0 = (x-4)(x+1)$

$\therefore x = 4 \quad or \quad x = -1$

Substitute $x = 4 \quad or \quad x = -1$ into equation 2

$y = (4) - 1$ or $y = (-1) - 1$

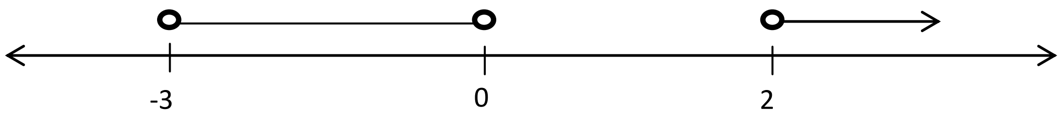
$y = 3 \ or \ y = -2$

$\therefore \ P(-1; -2) \quad and \quad Q(4; 3)$

Question 8

$x^3 + x^2 - 6x \geq 0$
$x(x^2 + x - 6) \geq 0$
$x(x+3)(x-2) \geq 0$
Critical values: $x = 0$; $x = -3$; $x = 2$

	$x < -3$	$x = -3$	$-3 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$x > 2$
x	$-$		$-$	0	$+$		$+$
$(x+3)$	$-$	0	$+$		$+$		$+$
$(x-2)$	$-$		$-$		$-$	0	$+$
	$-$		$+$		$-$		$+$



$x \in (-3; 0) \cup (2; \infty)$

Question 9

9.1 $2x^2 + 5x + 10 = 0$
$\Delta = b^2 - 4ac$
$\Delta = (5)^2 - 4(2)(10)$
$\Delta = 25 - 80$
$\Delta = -55$
$\therefore roots\ are\ non-real$

9.2 $x^2 - x = 6$
$\Delta = b^2 - 4ac$
$\Delta = (-1)^2 - 4(1)(-6)$
$\Delta = 1 + 24$
$\Delta = 25$
$\therefore roots\ are\ real\ and\ rational$

9.3 $4x^2 - 4x + 1 = 0$
$\Delta = b^2 - 4ac$
$\Delta = (-4)^2 - 4(4)(1)$
$\Delta = 16 - 16$
$\Delta = 0$
$\therefore roots\ are\ equal$

Question 10

$x^2 + k = kx + 2x$
$\therefore\ x^2 + k - kx - 2x = 0$
$\therefore\ x^2 + x(-k-2) + k = 0$
$\therefore\ \Delta = b^2 - 4ac$
$\therefore\ \Delta = (-k-2)^2 - 4(1)(k)$
$\therefore \Delta = k^2 + 4k + 4 - 4k$
$\therefore\ \Delta = k^2 + 4$
$\therefore\ k^2 \geq 0$ $(any\ value\ that\ is\ being\ squared\ is\ positive\ or\ equal\ to\ 0)$
$\therefore k^2 + 4 > 0$
$(by\ adding\ 4\ to\ the\ already\ positive\ value\ of\ k^2\ proves\ that\ the\ roots\ will\ be\ positive\ and\ greater\ than\ 0)$
$\therefore \Delta > 0$ $, therefore\ roots\ are\ real\ and\ unequal$

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