A Guide to Euclidean Geometry

Teaching Approach

Geometry is often feared and disliked because of the focus on writing proofs of theorems and solving riders. The focus of the CAPS curriculum is on skills, such as reasoning, generalising, conjecturing, investigating, justifying, proving or disproving, and explaining. As a result, learners should be discouraged from learning theorems off by heart and rather focus on developing their reasoning skills.

This section has been divided into three series of videos. The first series, *The Basics of Euclidean Geometry*, has three videos and revises the properties of parallel lines and their transversals. Learners should know this from previous grades but it is worth spending some time in class revising this. Euclidean Geometry requires the earners to have this knowledge as a base to work from.

The second series, *Triangles*, spends a large amount of time revising the basics of triangles. The videos investigate the properties of different triangles thoroughly giving the viewer a better understanding of the shape. The last few video lessons focus on similarity and congruency. These skills should have been covered in previous grades but it is worth spending time in class ensuring that the learners understand the concepts.

The last series, *Quadrilaterals*, investigates the properties of all the quadrilaterals in great detail.

These video lessons can be watched in any order. Once the section of work has been covered in class, why don't you spend some time doing the questions in the task video? These questions vary in difficulty and contain a combination of skills in each question.

Encourage your learners to make simple conjectures about triangles and then to test these ideas practically, by folding paper, measuring and constructing. Ensure that scissors, rulers, pencils, tracing paper and plain paper are available for this exercise.

Throughout the lessons, we remind viewers to write down any terminology they don't know and to add it to their glossary. This glossary should be kept in a safe place so that it can be used for studying. It is a good idea to designate the back of the book as the glossary.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

The Basics of Euclidean Geometry

1. **What is Euclidean Geometry?**

   This lesson introduces the concept of Euclidean geometry and how it is used in the real world today. This lesson also traces the history of geometry.

2. **Revising Lines and Angles**

   This lesson is a revision of definitions covered in previous grades. These include line segment, ray, straight lines, parallel lines and perpendicular lines. The lesson also teaches how to recognize and describe pairs of angles formed by perpendicular and parallel lines.

3. **Angles and Parallel Lines**

   In this lesson we explore the application of parallel lines and how we can use this information to calculate and find alternate, co-interior and corresponding angels.

Triangles

1. **An Introduction to Triangles**

   In this lesson we study the properties of triangles in terms of their sides and angles and classify them accordingly. We construct, measure and compare the lengths of the sides and the sizes of the angles.

2. **Investigating the Scalene Triangle**

   In this lesson we revise the properties of a scalene triangle and explore the terminology: Angle bisector, Point of concurrency, Perpendicular bisector, Median and Altitude.

3. **Investigating the Isosceles Triangle**

   In this lesson we revise the properties of an isosceles triangle and explore the terminology: Angle bisector, Point of concurrency, Perpendicular bisector, Median and Altitude.

4. **Investigating the Equilateral Triangle**

   In this lesson we revise the properties of an equilateral triangle and explore the terminology: Angle bisector, Point of concurrency, Perpendicular bisector, Median and Altitude.

5. **Theorems of Triangles**

   This lesson revises rules and theorems of triangles namely the sum of interior angles of a triangle and exterior angles of a triangle. We use constructions to learn about and show these theorems.

6. **Similar Triangles**

   In this lesson we define similarity and identify shapes that are similar, specifically focusing on similar triangles. Our approach is to test conjectures by investigating and measuring the angles and lines of the shapes. This lesson also links similarity to scale and ratio.

7. **Congruent Triangles**

   In this lesson we define congruency and identify congruent triangles. We begin by revising the four conditions that result in congruent triangles. This is done by investigating and measuring the angles and lines of the triangles so that learners can see for themselves.

8. **The Mid-point Theorem**

   In this lesson we investigate the line segments joining the midpoints of two sides of a triangle and introduce the new terminology that goes with this theorem. The investigation requires that learners do the measurements and constructions for themselves.

Quadrilaterals

1. **Revising Polygons**

   This lesson revises all the terminology and knowledge covered in previous grades regarding polygons.

2. **Investigating the Square**

   In this lesson we ask learners to investigate the properties of a square, by reflecting a right angled isosceles triangle twice. Remind learners to fill in all the information on the diagram as they are working through the investigation themselves.

3. **Investigating the Rhombus**

   In this lesson we revise the knowledge established so far. You may want to ask learners to create a rhombus by using two reflections of a right-angled scalene triangle and then describe the properties of the rhombus before showing the lesson.

4. **Investigating the Convex Kite**

   In this lesson we use an acute-angled scalene triangle and reflect it once to make a kite. We use congruency to prove that the diagonals of a convex kite are perpendicular to each other and that the vertex angles are bisected by the diagonal.

5. **Investigating the Concave Kite**

   Reflecting an obtuse-angled scalene triangle makes a concave kite. A concave kite has only one line of symmetry. Using an obtuse-angled isosceles triangle can also make a concave kite.

6. **Investigating the Rectangle**

   This lesson begins by investigating whether we can make a rectangle by reflection of a right angled scalene triangle, and we find that only rotation can produce a rectangle. We see that rotation always occurs around a fixed point; in this case, it must be the midpoint of the hypotenuse.

7. **Investigating the Parallelogram**

   In this lesson we rotate an obtuse-angled scalene triangle to make a parallelogram. As with convex kites, we use congruent triangles to prove the properties of the diagonals.

8. **Investigating the Trapezium**

   In this lesson, we create a trapezium by reflecting an isosceles right-angled triangle twice. We discover that the only properties of a trapezium are that one pair of opposite sides is parallel and the co-interior angles are supplementary.

9. **Comparing all Quadrilaterals**

   In this lesson we review all the quadrilaterals we have done in this series. We use a quiz to consolidate the properties of quadrilaterals. This also gives learners a chance to compare the properties of these shapes with each other.

Resource Material

The Basics of Euclidean Geometry		
1. What is Euclidean Geometry?	http://www.math.toronto.edu/mathnet/questionCorner/euclidgeom.html	This website defines Euclidean Geometry.
2. Revising Lines and Angles	http://www.mathsisfun.com/geometry/parallel-lines.html	This website explains the concepts parallel lines and pairs of angles.
	http://www.ezschool.com/EZSheets/Geometry/Lines_Angles/index.html	This is a premier educational portal that provides students with resources on Mathematics from counting by using objects to multiplication.
3. Angles and Parallel Lines	http://www.mathsteacher.com.au/year8/ch09_geometry/03_parallel/lines.htm	This website looks at angles associated with parallel lines
Triangles		
1. An Introduction to Triangles	http://www.education.com/study-help/study-help-geometry-triangles/	This website provides students with study guides which have explanation, example problems, and practice problems with solutions to help you learn triangles for geometry.
2. Investigating the Scalene Triangle	http://www.mathopenref.com/scalene.html	This is an all in one package on scalene triangles that defines scalene triangles and provides facts about them.
3. Investigating the Isosceles Triangle	http://www.mathopenref.com/isosceles.html	This website provides information on isosceles triangles.
4. Investigating the Equilateral Triangle	http://www.mathopenref.com/equilateral.html	This is a great website to get the basics on equilateral triangles
5. Theorems of Triangles	http://www.sparknotes.com/math/geometry2/theorems/section1.html	This website gives a brief overview on theorems of triangles
6. Similar Triangles	http://www.mathopenref.com/similartriangles.html	Refer to this website to learn more about when triangles are considered similar.
	http://www.kutasoftware.com/freeige.html	This website is a test and worksheet generator for Math teachers.
7. Congruent Triangles	http://www.mathopenref.com/congruenttriangles.html	This website defines when a triangle is considered congruent.
	http://www.kutasoftware.com/freeige.html	This is a test and worksheet generator for Math teachers
8. The Mid-point Theorem	http://m.everythingmaths.co.za/grade-10/11-geometry/11-geometry-04.cnxmlplus	A chapter from Everything Maths textbook on the midpoint theorem:

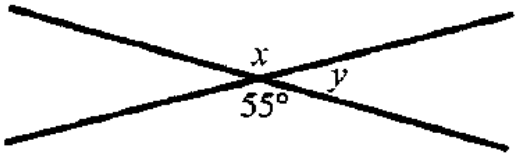
Quadrilaterals		
1. Revising Polygons	http://www.mathsisfun.com/geometry/polygons.html	A useful site to explore polygons.
	http://www.math-play.com/types-of-poligons.html	Learn about all the different types of polygons on this site.
2. Investigating the Square	http://www.mathopenref.com/square.html	Learn more about squares on this site.
3. Investigating the Rhombus	http://www.mathsisfun.com/geometry/rhombus.html	For everything there is to know about rhombus refer to this site.
4. Investigating the Convex Kite	http://www.mathopenref.com/kite.html	Learn about convex kites here.
5. Investigating the Concave Kite	http://www.mathopenref.com/kite.html	This website provides a fun and interactive way of learning about the concave kite
6. Investigating the Rectangle	http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/rectangle.php	This website provides an overview on rectangles.
7. Investigating the Parallelogram	http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/	This website provides a fun and interactive way of learning about parallelograms
8. Investigating the Trapezium	http://www.coolmath.com/reference/trapezoids.html	Note on the properties of a trapezoid:
9. Comparing all Quadrilaterals	http://www.thatquiz.org/tq/previewtest?G/H/F/S/6337130804980	Test your knowledge on quadrilaterals on this website.
	http://ebookbrowse.com/notes-blank-quadrilaterals-flow-chart-pdf-d296283022	Here you will find a blank quadrilateral flow chart where you can take notes.

Task

Question 1

Solve for the unknown angles.

1.1.



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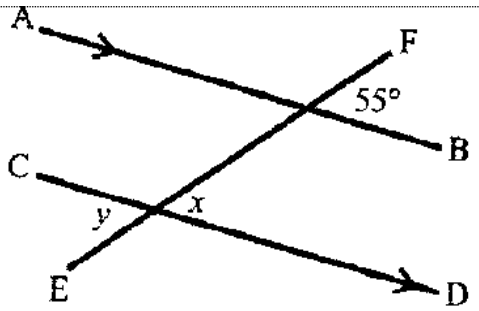

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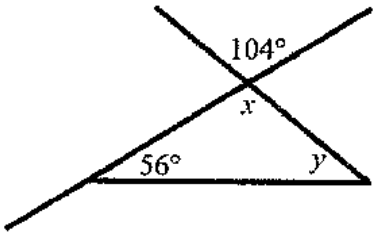
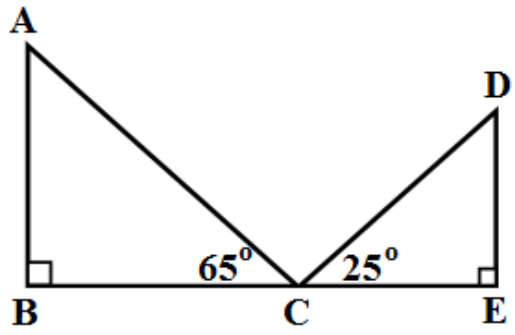


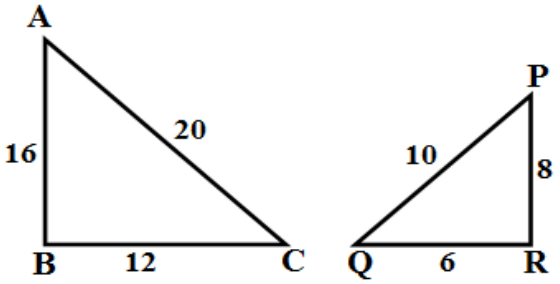
Question 2

State with reasons whether each triangle is similar or congruent.

2.1.

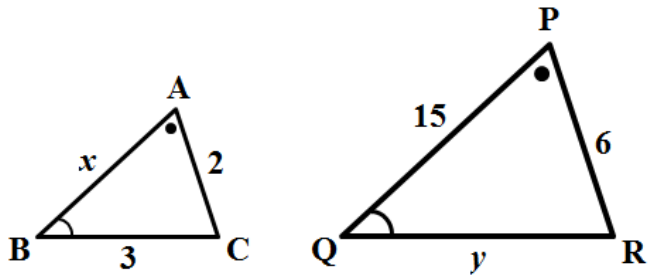


2.2.



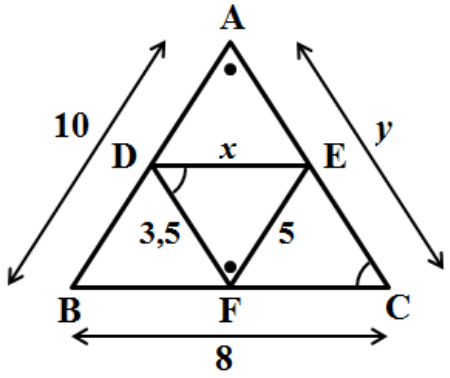
Question 3

Given that the following triangle are similar, determine the values of x and y



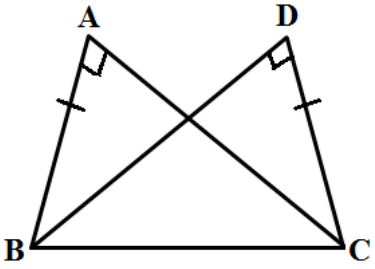
Question 4

ΔABC is similar to ΔDEF. Calculate x and y.



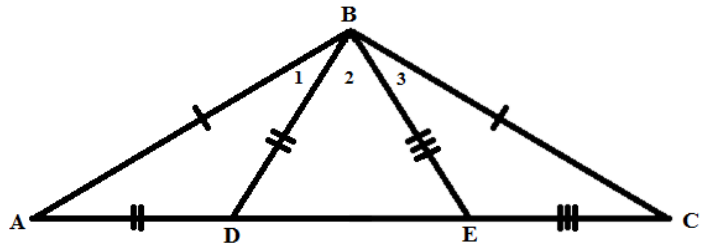
Question 5

Prove that triangle ABC is congruent to triangle DCB



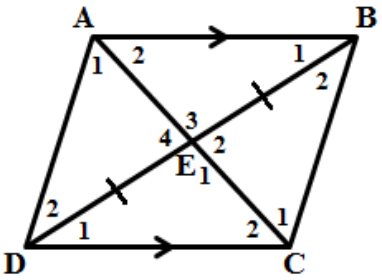
Question 6

Prove that triangle ABD is congruent to triangle CBE



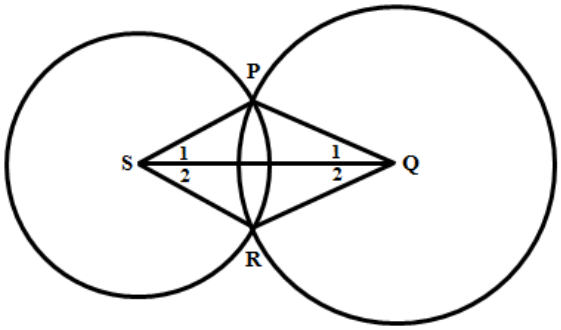
Question 7

Prove that quadrilateral ABCD is a parallelogram



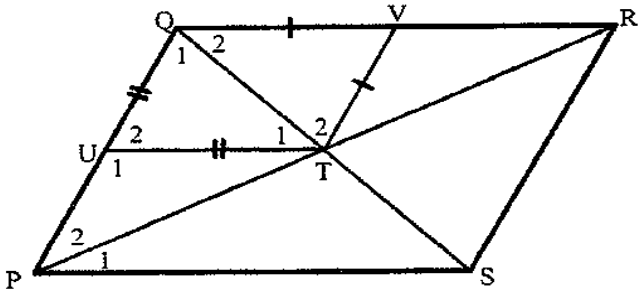
Question 8

Given are two circles that intersect at P and R. S and Q are centres of the respective circles. Prove that PQRS is a kite.

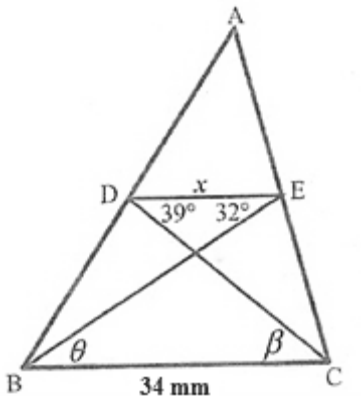


Question 9

PQRS is a rhombus with its diagonals intersecting at T. TV =TQ and TU = QU. Prove that TUQV is a rhombus.



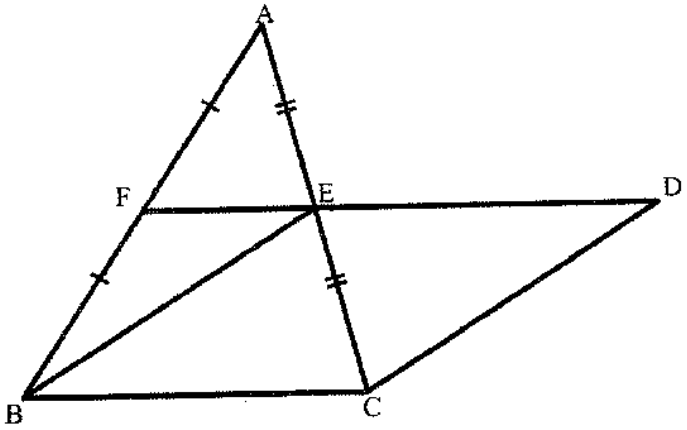
Question 10



In Δ ABC points D and E are midpoints of AB and AC respectively.
BC = 34mm
$\angle$ EDC is 39^0
$\angle$ DEB is 32^0.
Using the given information, determine the values of x, β and θ

Question 11

In Δ ABC, F and E are the midpoints of AB and AC respectively. ED = 2FE. Prove that BCDE is a parallelogram.



Task Answers

Question 1

1.1.

$x = 55°$ Vert. opp angles are equal

$y = 180° - 55°$ Adj. angles on a straight line

$\therefore y = 125°$

1.2.

$x = 55°$ Corresponding angles are equal AB//CD

$y = 55°$ Vertical opp. angles are equal

1.3.

$x = 104°$ Vert. opp. angles are equal

$y = 180° - 56° - 104°$ Sum of angles in a triangle

$\therefore y = 20°$

Question 2

2.1.

$\hat{A} = 180° - 90° - 65°$ sum of angles in a triangle

$\therefore \hat{A} = 25°$

$\therefore \hat{A} = DCE$

$\hat{B} = \hat{E}$ Given

$\hat{D} = 180° - 90° - 25°$ sum of angles in a triangle

$\therefore \hat{D} = 65°$

$\therefore \hat{D} = BCA$

$\therefore \Delta ABC \,|||\, \Delta CED \quad (AAA)$

2.2.

$\frac{AB}{PR} = \frac{AC}{PQ} = \frac{BC}{QR} = \frac{2}{1}$ Given

$\frac{16}{8} = \frac{20}{10} = \frac{12}{6}$

$\frac{2}{1} = \frac{2}{1} = \frac{2}{1}$

$\therefore \Delta ABC \,|||\, \Delta PRQ$

Question 3

3.1.

$\frac{x}{15} = \frac{2}{6}$ ABC III PQR

$6x = 30$

$\therefore x = 5$

$\frac{y}{3} = \frac{6}{2}$ ABC III PQR

$2y = 18$

$\therefore y = 9$

3.2.

$y = 7 \quad x = 4$

Question 4

4.1

In ∆ABC and ∆BCD

$\hat{A} = \hat{D}$ Given

AB= DC Given

BC =BC Common

∴ $\Delta ABC \equiv \Delta DCB$ RHS

4.2

In ∆ABD and ∆CBE

AB = BC Given

$\hat{A} = \hat{C}$ Isosceles triangle base angles equal

BE = EC Given

∴ $\hat{C} = \widehat{B_3}$ Isosceles triangle

BD = AD Given

∴ $\hat{A} = \widehat{B_1}$ Isosceles triangle

∴ $\hat{A} = \widehat{B_1} = \hat{C} = \widehat{B_3}$

∴ $\Delta ABD \equiv \Delta CBE$ SAA

Question 5

Note: There are two parts to this proof. Since AB is given as being parallel to CD it remains to be proven that it is equal as well. To do that it is helpful to show that ∆ABE ≡ ∆ CED

DE = EB Given

$E_1 = E_3$ Vert. opp. angles are equal

$B_1 = D_1$ Alt. angles are equal, AB//DC

∴ $\Delta ABE \equiv$ ∆CED SAA

∴AB =DC

∴ABCD is a parm, one pair of sides are equal and parallel.

Question 6

SP = SR Radii

PQ = QR Radii

PQRS is a kite Two pairs of adjacent sides are equal.

Question 7

Note: In order to prove that QUTV is a rhombus, it is necessary to prove that it's a parallelogram with equal adjacent sides.

$\widehat{Q_1} = \hat{Q}_2$ Diagonals of rhombus PQRS bisect angles

$\widehat{Q_1} = \hat{T}_1$ Isosceles triangle base angles are equal

$\widehat{Q_2} = \hat{T}_2$ Isosceles triangle base angles are equal

∴ $Q_2 = T_1$ *and* $Q_1 = T_2$

∴ QU = QV and VT = UT

∴ QV//UT and QU//VT

∴ $QUTV$ is a rhombus Parm with adjacent sides equal

Question 8

DE = $\frac{1}{2}$ BC Midpoint Theorem

$\therefore x = 17mm$ Midpoint Theorem

$\theta = 32°$ alt angle equal DE//BC

$\beta = 39°$ alt angle equal DE//BC

Question 9

AF = FB and AE = EC	Given
∴ FE//BC and BC= 2FE	Midpt theorem
$\therefore ED//BC$	
ED = 2FE	Given
∴ ED = BC	
∴ BCDE is a parm	Pair of opp. sides are equal and parallel.

Question 10

Statement	Reason
AD = DB and AE = EC	Given D and E are mid pts.
∴ DE = $\frac{1}{2}$ BC	Mid pt Theorem
∴ DE = $x = 17mm$	
DE II BC	Mid pt Theorem
∴ $\theta = 32°$	Alt angles, DE II BC
∴ $\beta = 39°$	Alt angles, DE II BC

Question 11

Statement	Reason
AF = FB and AE = EC	Given F and E are mid pts.
BC = 2FE	Mid pt Theorem
ED = 2FE	Given
∴ BC = ED	
FE II BC	Mid pt Theorem
∴ FD II BC	
∴BCDE is a parm	Opp. Sides equal and parallel

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