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SESSION 8: TRIGONOMETRIC FUNCTIONS

KEY CONCEPTS:

- **Graphs of Trigonometric Functions**
  y = sin θ
  y = cos θ
  y = tan θ

- **Properties of Graphs**
  Shape
  Intercepts
  Domain and Range
  Minimum and maximum values
  Period and amplitude

- **Transformations**
  Effects of changing amplitude
  Shifting the function up / down.
  Shifting the function up / down.

TERMINOLOGY

Amplitude the maximum distance from the point of rest

Domain the input values of a function. These are the angle values for trig functions

Range the output values of a function. These are the values of the ratios for trig functions

Minimum value the smallest value in the range

Maximum value the largest value in the range

Period the set of x-values (angles) for which the graph is not repeated

Intercept the points where the graph cuts the axes

Parent function the simplest form of the trigonometric function and used to generate families of functions by changing the a and q values in the general equation e.g $f(x) = a \sin x + q$ The value of a =1 and q=0 for the parent function.

Asymptote A value of the domain (angle) for which the ratio (function) is undefined. Asymptotes are indicated by a dotted line on the graph.

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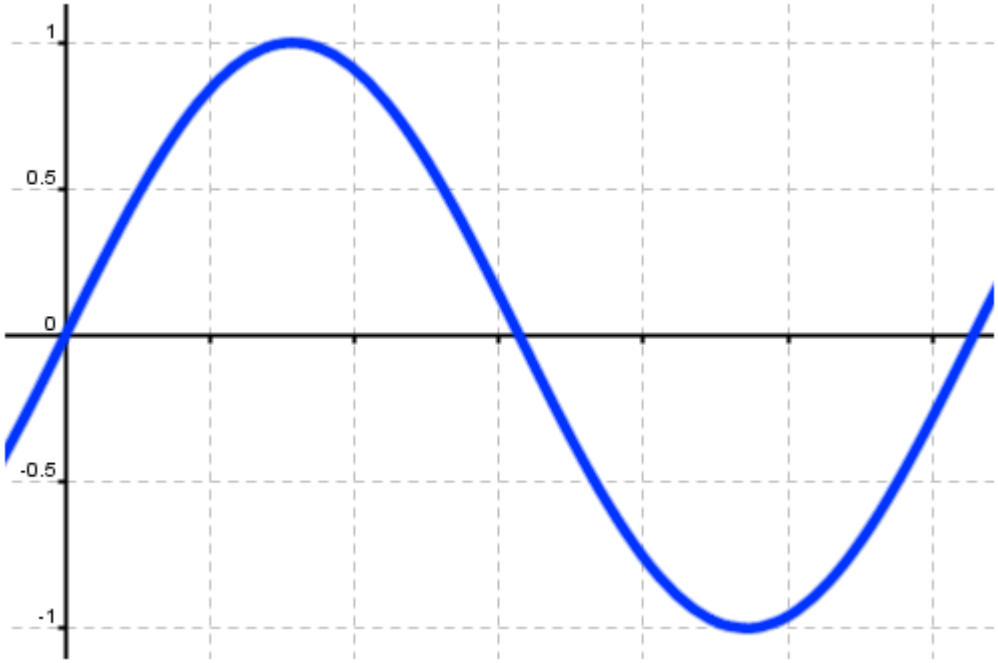
X-PLANATION

We can tabulate the values of the angle and the value of the trigonometric ratios and so define trigonometric functions. We also plot the points of these functions to generate different graphs of the primary trigonometric ratios. We start with the simplest function, called the parent function and show how this function can be transformed to generate a family of other functions.

Sine Function:

Parent function: f(x) = sin x for the domain: $[0^0; 360^0]$



Shape:	Wave-like shape, starting at the origin
Intercepts:	y-intercept = 0
	x-intercept = 0^0, 180^0, 360^0 (every 180^0 starting at 0^0)
Domain:	The domain is usually limited to the interval $[0^0; 360^0]$
	Infinite angles are possible as a line centred at the origin on the Cartesian plane can be rotated many times. Rotating the line anti-clockwise gives positive angles and rotating clockwise gives negative angles.
Period:	This corresponds to one rotation. The sine function repeats itself every 360^0.
Range:	Minimum value: -1 when the angle x is 270^0 or -270^0
	Maximum value: 1 when the angle x is 90^0 or -90^0
	[-1; 1]

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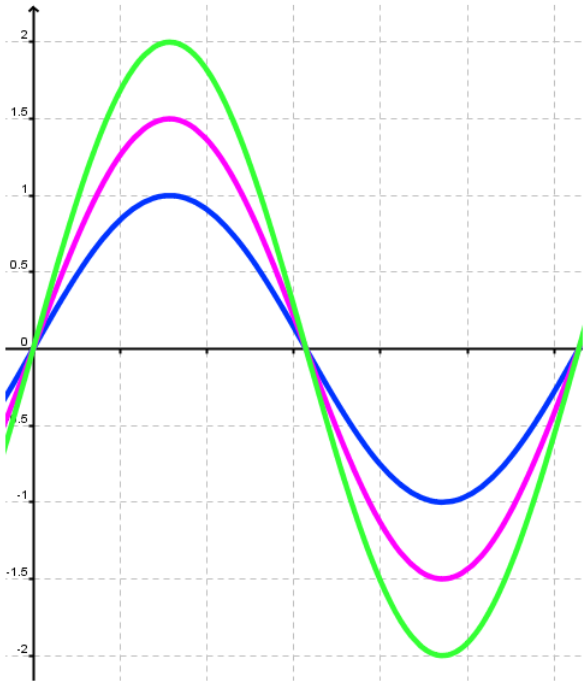
Amplitude: For the parent function the amplitude is 1. It is half the range.

General form $f(x) = a \sin x + q$

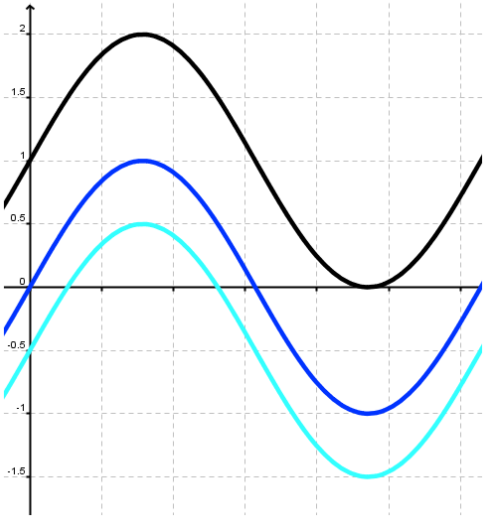
a – amplitude. For the parent function a = 1

The bigger the value of a the bigger the maximum value will be. The graph is stretched away from the x-axis (rest position). Changing a does not change the x-intercepts when q =0.



q – rest position: For the parent function q = 0

this value shifts the whole graph vertically up when it is positive and down when it is negative. The q value changes the position of the rest position and will change the value of the intercepts



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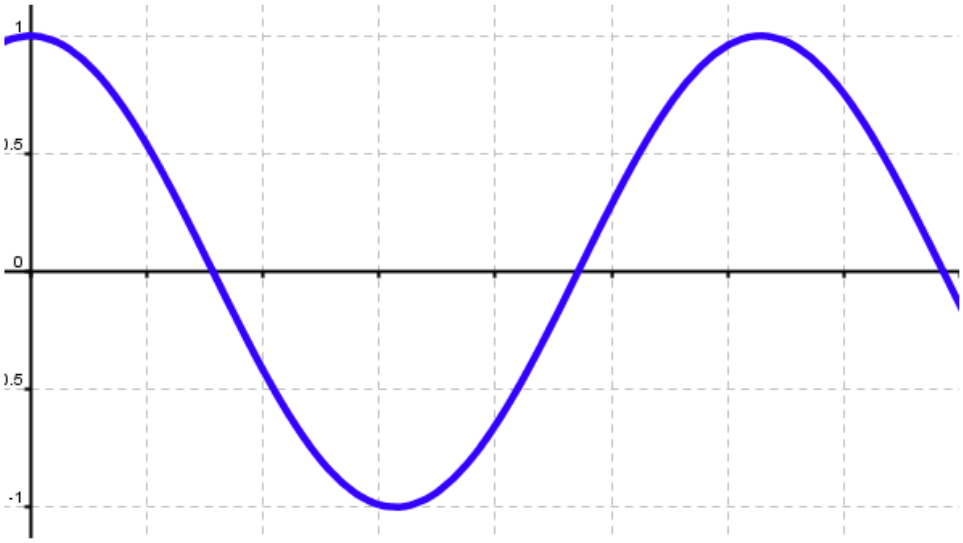
Cosine Function

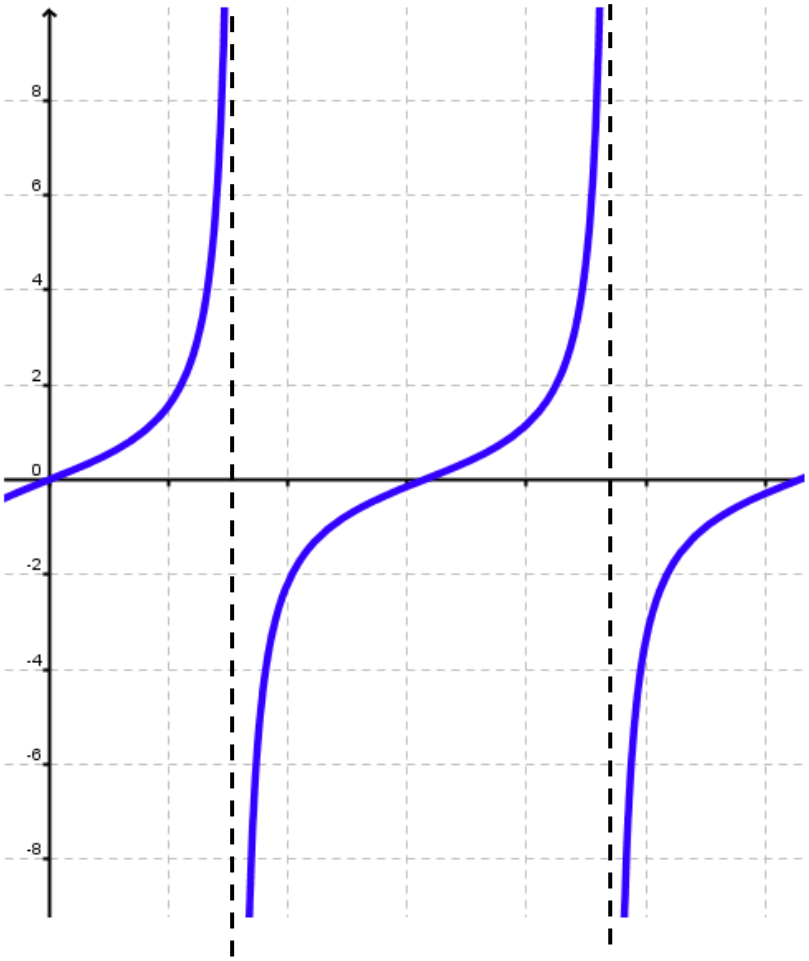
Parent function: f(x) = cos x for the domain: [0^0; 360^0]



Shape:	Wave-like shape but when x = 0^0 the graph is a 1
Intercepts:	y-intercept = 1
	x-intercept = 90^0, 270^0, (every 180^0 starting at 90^0)
Domain:	The domain is usually restricted to the interval [0^0; 360^0] or [-360^0; 360^0]
Period:	This corresponds to one rotation. The cosine function repeats itself every 360^0.
Range:	Minimum value: -1 when the angle x is 90^0 or -90^0
	Maximum value: 1 when the angle x is 0^0 ; 360^0 and -360^0
	[-1; 1]
Amplitude:	For the parent function the amplitude is 1. It is half the range.
General form	f(x) = a cos x + q
	a – amplitude. For the parent function a = 1
	The bigger the value of a the bigger the maximum value will be. The graph is stretched away from the x-axis (rest position). Changing a does not change the x-intercepts when q =0.
	q – rest position: For the parent function q = 0
	This value shifts the whole graph vertically up when it is positive and down when it is negative. The q value changes the position of the rest position and will change the value of the intercepts

Tangent Function

Parent function: f(x) = tan x for the domain: $[0^0; 360^0]$



Shape:	Not wave-like shape. Long thin curve that is repeated
Intercepts:	y-intercept = 0
	x-intercept = 0^0, 180^0, (every 180^0 starting at 0^0)
Domain:	The domain is usually restricted to the interval $[0^0; 360^0]$ or $[-360^0; 360^0]$
	For x = $\pm 90^0$ and $\pm 270^0$, the function is undefined.
Period:	The tangent function repeats itself every 180^0, starting at -90^0 to 90^0
Range:	$(-\infty;\infty)$
	The minimum and maximum occur at the asymptotes at $\pm 90^0$ and $\pm 270^0$. (Every 180^0 starting at 90^0)
Amplitude:	Since the tangent function is not a wave like graph it do not have an amplitude. However, for the parent function when x = 45^0, the value of the function is 1
General form	f(x) = a tan x + q

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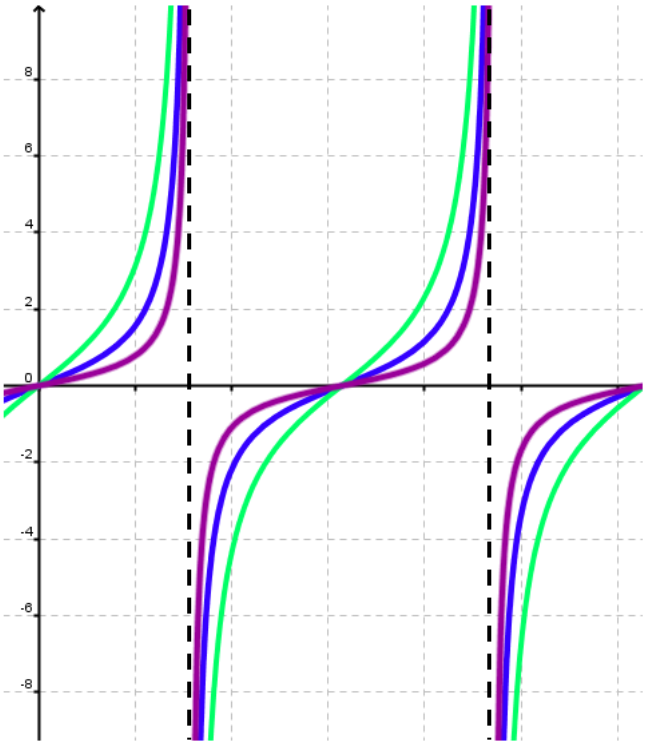
For the parent function a = 1

a > 1 stretches the graph away from the x-axis

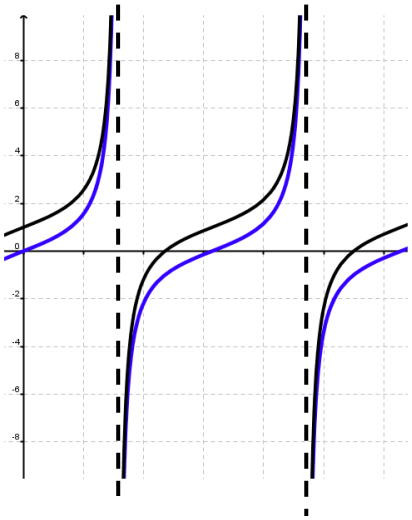
a < 1 pulls the graph closer to the x-axis



Changing a does not change the x-intercepts or the asymptotes when q =0.

q – rest position: For the parent function q = 0

This value shifts the whole graph vertically up when it is positive and down when it is negative. The q value changes the position of the rest position and will change the value of the intercepts but not the asymptotes.



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X-AMPLE QUESTIONS:

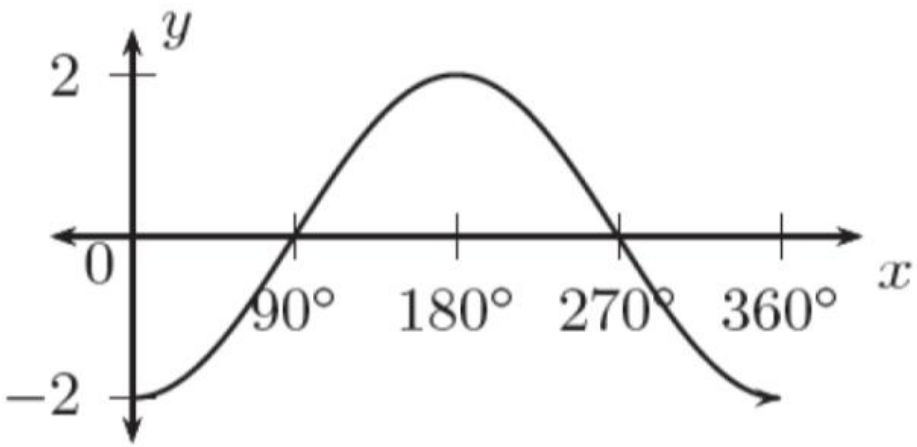
Question 1:
Sketch the graph of f(θ) = 2 sin θ + 3 for $\theta \in [0^o; 360^o]$

Question 2:
Sketch the graph of f(θ) = 2 cos θ + 3 for $\theta \in [0^o; 360^o]$

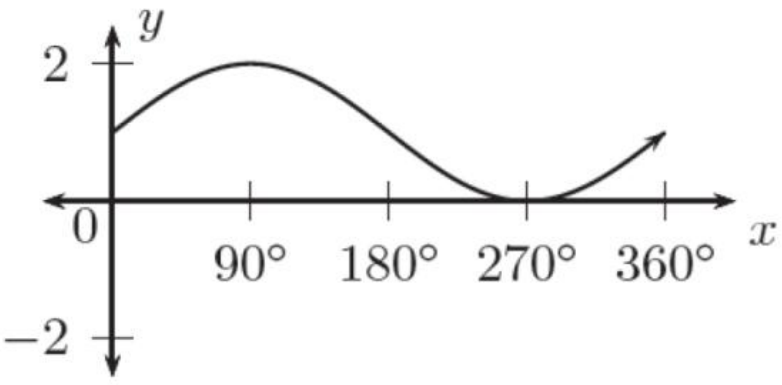
Question 3:
Sketch the graph of y = 2 tan θ + 1 for $\theta \in [0^o; 360^o]$

X-ercise
Study the following trigonometric functions and determine the equations:

1.



2.



Solution:
1. y= -2cosx
2. y = sinx +1