

## ANALYTICAL GEOMETRY

09 JUNE 2014



### Lesson Description

In this lesson we:

- Work through a number of examples to focus on the major key concepts in Analytical Geometry:
  - 1 Gradient; Parallel Lines; Perpendicular Lines
  - 2 Equation of a Line
  - 3 Distance of a Line Segment and Midpoint of a Line Segment.
  - 4 Inclination
  - 5 Properties of Geometrical Figures



### Summary

(From Grade 11, *Clever Keeping Maths Simple*, Macmillan, Page 91)

1. <b>The distance formula</b> Used to find the length of a line segment	$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$ or $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
2. <b>The midpoint formula</b> M is the midpoint of line segment PQ	$x_M = \frac{x_P + x_Q}{2}; y_M = \frac{y_P + y_Q}{2}$
3. <b>The gradient of a straight line</b>	
3.1 If given the coordinates of two points on the straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
3.2 If given two parallel straight lines:	$m_1 = m_2$
3.3 If given two perpendicular straight lines:	$m_1 \times m_2 = -1$ or $m_1 = -\frac{1}{m_2}$
3.4 If given the inclination angle:	$m = \tan \theta$
4. <b>The equation of a straight line</b> You always need to know: <ul style="list-style-type: none"> <li>• the gradient</li> <li>• the coordinates of a point on the line</li> </ul>	$y - y_1 = m(x - x_1)$



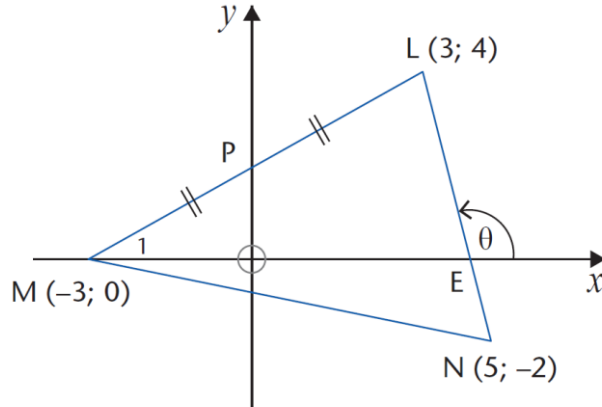
**Test Yourself**

**Question 1**

(From Grade 11, *Clever Keeping Maths Simple*, Macmillan, Pg 109, Revision Exercise, Question 10)

The diagram below shows the points  $L (3; 4)$ ,  $M (-3; 0)$  and  $N (5; -2)$ .

$P$  is the midpoint of  $ML$  and  $LN$  cuts the  $x$ -axis at  $E$ .  $\theta$  is the inclination of  $LN$ .



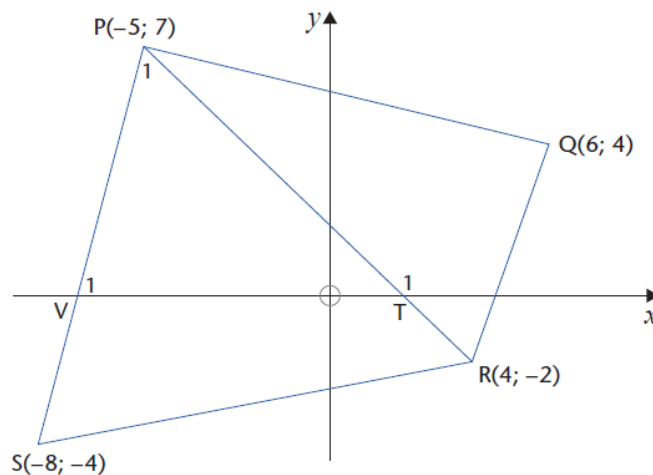
Calculate:

- The length of  $MN$ , giving your answer in simplest surd form
- The coordinates of  $P$ , the midpoint of  $ML$
- The equation of  $LN$
- The value of  $\theta$ , correct to one decimal place
- The inclination of  $ML$ , correct to one decimal place
- The size of  $\hat{M}\hat{L}\hat{N}$ .

**Question 2**

(From Grade 11, *Clever Keeping Maths Simple*, Macmillan, Page 104, Exercise 4.6, Question 2)

$P(-5; 7)$ ,  $Q(6; 4)$ ,  $R(4; -2)$  and  $S(-8; -4)$  are the vertices of quadrilateral  $PQRS$ .



- Prove that  $PS \perp PQ$ .
- Determine the equation of  $PR$ .
- Calculate the coordinates of  $S$ , the  $y$ -intercept of  $PR$ .
- Calculate the size of  $\hat{T}_1$ .

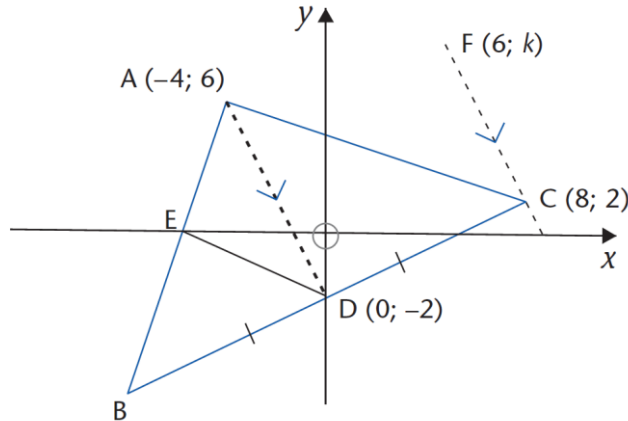


## Improve your Skills

### Question 1

(From Grade 11, *Clever Keeping Maths Simple*, Macmillan, Pg 108, Revision Exercise, Question 8)

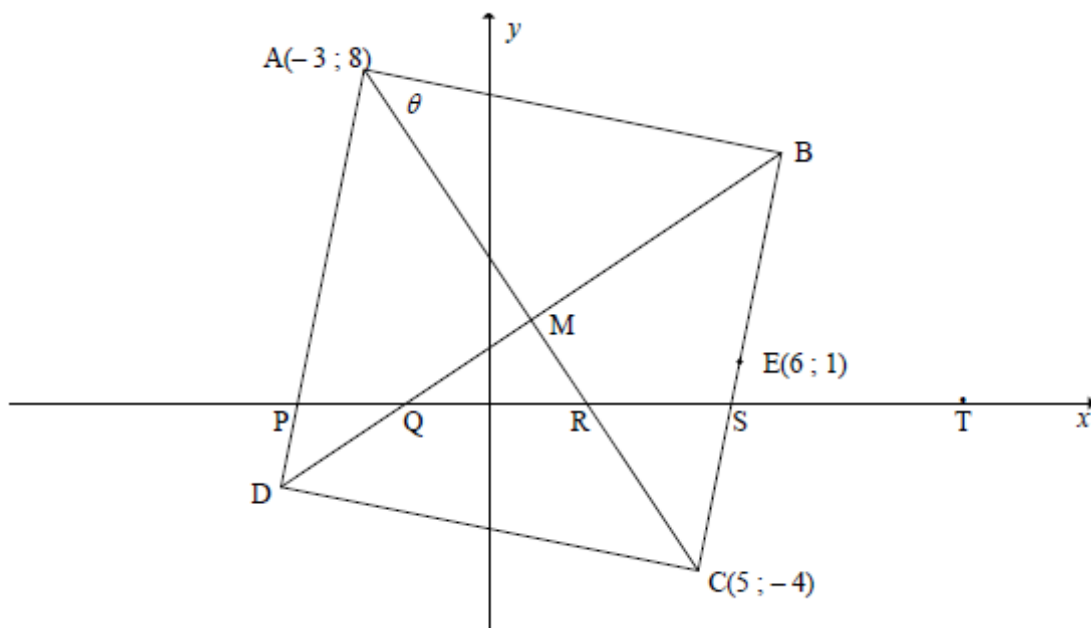
In  $\triangle ABC$  the coordinates of the vertices A and C are  $(-4; 6)$  and  $(8; 2)$  respectively. D  $(0; -2)$  is the midpoint of BC and E is the midpoint of AB. F is the point  $(6; k)$ , with  $FC \parallel AD$ .



- Determine the coordinates of B.
- Give the coordinates of E, the midpoint of AB.
- Prove that  $ED \parallel AC$ .
- Show analytically that  $AC = 2(ED)$ .
- Prove that  $AD \perp BC$ .
- Calculate the value of  $k$  if  $FC \parallel AD$ .
- Calculate the length of BC, giving your answer in simplest surd form.
- Determine the area of  $\triangle ABC$ .

### Question 2

ABCD is a rhombus with A  $(-3; 8)$  and C  $(5; -4)$ . The diagonals of ABCD bisect each other at M. The point E  $(6; 1)$  lies on BC.



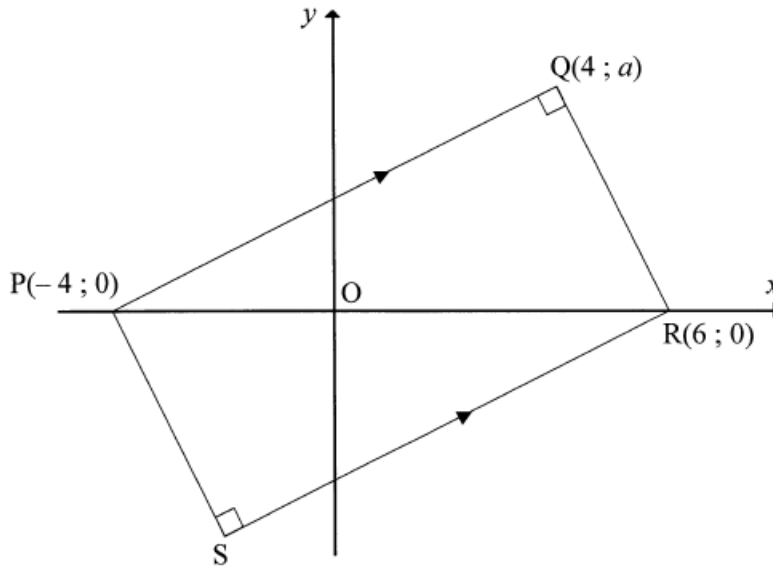
notes for...

- Calculate the coordinates of M.
- Calculate the gradient of BC.
- Determine the equation of the line AD in the form  $y = mx + c$ .
- Determine the size of  $\theta$ , that is  $\widehat{BAC}$ . Show ALL calculations.

**Question 3**

In the diagram below., PQRS is a rectangle with vertices  $P(-4 ; 0)$ ,  $Q(4 ; a)$ ,  $R(6 ; 0)$  and S.

Q lies in the first quadrant.



- Show that  $a = 4$
- Determine the equation of the straight line passing through the points S and R in the form  $y = mx + c$ .
- Calculate the coordinates of S.
- Calculate the length of PR.