REVISION: MOTION & WORK 19 MARCH 2013

Lesson Description

In this lesson we revise how to:

- Apply the Law of conservation of momentum
- Calculate work done and power

Key Concepts

Terminology

System: the collection of objects in question.

Momentum: the amount of motion a body has due to its mass and velocity.

Conservation of Momentum: the total linear momentum of an isolated will remain the same.

Impulse: the change in a object's momentum due to a force being exerted on it for a time.

Collision: the rapid striking of two or more objects together.

Explosion: the sudden, forceful separation of objects.

Elastic collision: collision where kinetic energy is conserved.

Inelastic collision: collision where kinetic energy is lost.

Momentum

The momentum of an object is the amount fo motion it has. It is calculated by multiplying the object's mass (kg) and its velocity ($m{\cdot}s^{-1}$).

$$p = mv$$

The answer is measured in **$kg{\cdot}m{\cdot}s^{-1}$** and **is a vector**. This means that direction plays an important role in an object's mometum.

Conservation of Momentum

As mentioned before, impulse is the way that objects trade mometum on one another, allowing the total mometum of the system to remain constant.

When approaching a question of this type, it is important to note that **direction plays a key role**.

The idea is as follows:

- The total momentum of all objects is added up before the collision
- The total mometum of all the object is added up after the collision
- These two totals are made equal and any unknown values are found.

There are some important steps in approaching these questions. We will use a simple example later to show you how they work.

Step 1: Diagram and Direction.

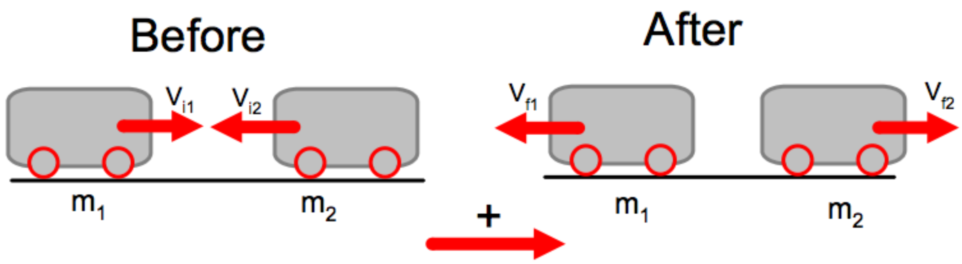
As with any good vector question, diagram and direction is the first step. Quickly draw a sketch and indicate which direction will be your positive direction. It may be chosen by the question but **stick to it!** Don't ever change the answers of your calculations to suit your personal feelings or change positive directions during the question.

You will need to have a diagram for the objects **before** and **after**.

Step 2: Conservation Equation.

Now, we are ready to set up our conservation of momentum equation. How we do this depends on how many objects there were before and after the collision.

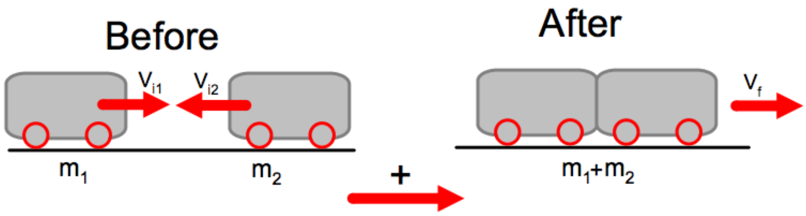
Note: each object needs its own "**mv**" term in the equation, see below for ideas.



$$p_i = p_f$$

$$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$$

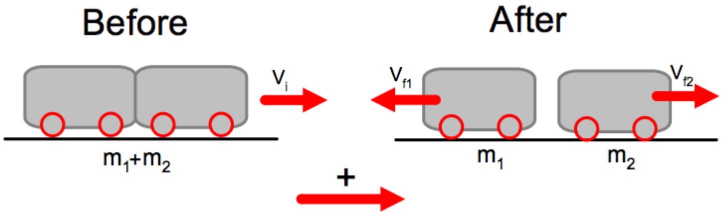
Notice how each separate object has its own "**mv**". Here, two objects collided and left seperately so there were two objects before and two objects after. So the final equation has four pieces.



$$p_i = p_f$$

$$m_1v_{i1} + m_2v_{i2} = (m_1+m_2)v_f$$

Here, two objects collided and combined so there were two objects before and one object after. So the final equation has three pieces.



$$p_i = p_f$$

$$(m_1+m_2)v_f = m_1v_{f1} + m_2v_{f2}$$

Here, one object exploded and seperated so there was one object before and two objects after. So the final equation has three pieces.

Elastic Collisions

We know that momentum is conserved in a collision but what about the other quantity of motion, kinetic energy?

When a collision conserves kinetic energy we say that the collision is **elastic**. If kinetic energy afterwards does not equal the amount that there was before, the collision was **inelastic**. Often we have to prove this using calculations of $K = \frac{1}{2}mv^2$ for all objects before an after.

Questions

DOE March 2011 Question 4

Two shopping trolleys, X and Y, are both moving to the right along the same straight

line. The mass of trolley Y is 12 kg and its kinetic energy is 37,5 J.

1) Calculate the speed of trolley Y.

This is a simple energy calculation:

$$K = \frac{1}{2}mv^2$$

$$37{,}5 = \frac{1}{2}(12)v^2$$

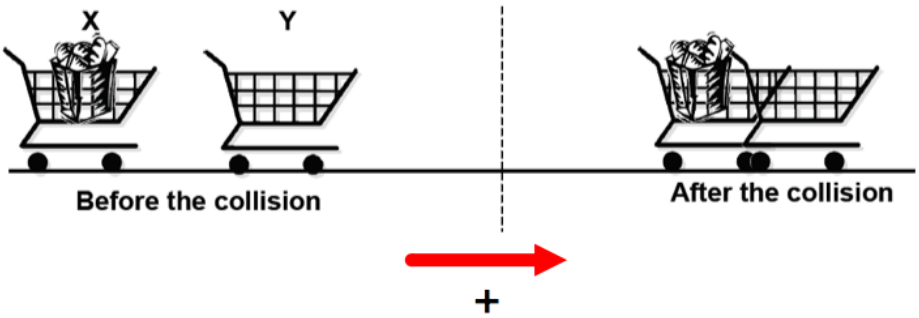
$$v = 2{,}5 \text{ m·s}^{-1}$$

Now comes the momentum question:

Trolley X of mass 30 kg collides with trolley Y and they stick together on impact. After the collision, the combined speed of the trolleys is 3,2 m·s^{-1}. (Ignore the effects of friction.)

2) Calculate the speed of trolley X before the collision.

Step 1: Diagram and Direction



Notice how the direction that we consider positive is marked with an arrow and a "plus" sign? This is the original direction of Trolley X.

Step 2: Conservation of momentum calculation

$$p_i = p_f$$

$$m_1 v_{i1} + m_2 v_{i2} = (m_1 + m_2) v_f$$

$$(30)v_{i1} + (12)(2,5) = (30 + 12)(3,2)$$

$$v_{i1} = 3,48 \text{ m·s}^{-1} \text{ in original direction of motion.}$$

During the collision, trolley X exerts a force on trolley Y. The collision time is 0,2 s.

3) Calculate the magnitude of the force that trolley X exerts on trolley Y.

Here, we have an indication that time is playing a role. The only equation using time and momentum is:

$$F\Delta t = mv_f - mv_i$$

$$F(0,2) = (12)(3,2) - (12)(2,5)$$

$$F = 42 \text{ N in original direction of motion}$$

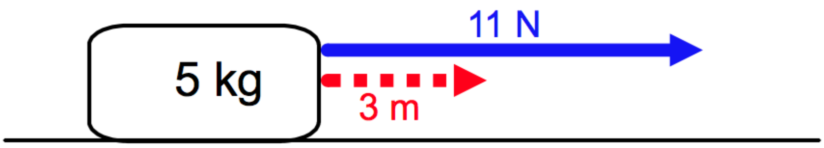
Key Concepts

Work

Work is done when a force moves an object over a distance.

It is very important to note that only when force and displacement act in the same plane, is work done.

However, from Grade 11, we know that forces can act parallel to the plane or perpendicularly to it. So let's use the example of an object pulled along by a force of 11 N over a distance of 3 m. For this example – let us have the force acting in the same direction as the displacement.



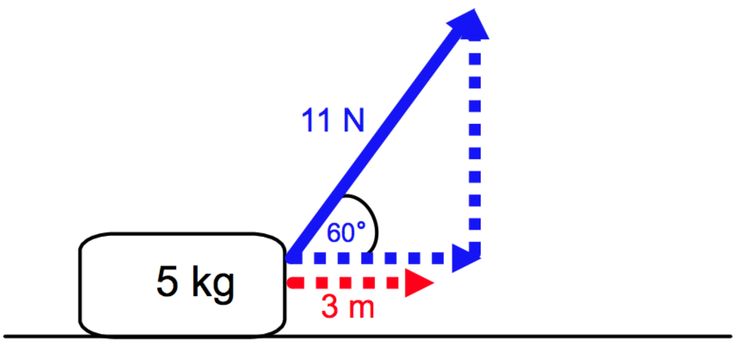
To calculate the work done is simple, we multiply the force applied and the displacement.

$W = F_{\parallel}\Delta x$

$W = (11)(3)$

$W = 33\ J$

What about if the force is pulled at 60° to the surface? Now we'd need to find out how much of the force is parallel to the surface – now we work with components. We need the parallel component of the applied force:



$F_{\parallel} = F\cos\theta$

$F_{\parallel} = 11\cos 60°$

$F_{\parallel} = 5{,}5\ N$ parallel to surface

Now, we can multiply the force parallel to the displacement and get the work done:

$W = F_{\parallel}\Delta x$

$W = (5{,}5)(3)$

$W = 16{,}5\ J$

The Work-Energy Theorem

Remember:

- When a force is applied to an object while it moves, work is done.
- When there is a resultant/net force on the object, that object will accelerate
- The "extra" force, or net force does work that results in a change in the object's kinetic energy.

So, the equation looks like the one below:

$$W_{net} = \Delta K = \Delta E_k$$

The W_{net} is simply, the work done by all the forces acting on the object:

- If the force increases the energy of the object, by acting in the same direction as the movement, W_{net} becomes larger;
- If the force acts against the object moving, W_{net} becomes smaller.

Power

Power is the rate at which work is done. Sometimes it is described as the "**rate of energy transfer**".

$$P = \frac{W}{\Delta t}$$

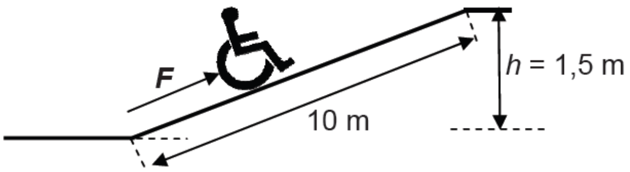
Questions

Question 2

(Adapted from DOE Nov 2009, Question 5, Paper 1)

John applies a force F to help his friend in a wheelchair to move up a ramp of length 10 m and a vertical height of 1,5 m, as shown in the diagram below. The combined mass of his friend and the wheelchair is 120 kg. The frictional force between the wheels of the wheelchair and the surface of the ramp is 50 N. The rotational effects of the wheels of the wheelchair may be ignored.

The wheelchair moves up the ramp at constant velocity



a.) What is the magnitude of the net force acting on the wheelchair as it moves up the ramp? Give a reason for your answer. (2)

b.) What is the magnitude of the net work done on the wheelchair on reaching the top of the ramp? (1)

c.) Calculate the following:

  i. Work done on the wheelchair by force F (5)

  ii. The magnitude of force F exerted on the wheelchair by John (4)

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