

## PROJECTILE MOTION: CONSERVATION OF MOMENTUM

19 FEBRUARY 2013

### Lesson Description

In this lesson we:

- Learn that an object's momentum is the "amount of motion" it has due to its mass and velocity.
- Show that momentum during collisions and explosions is conserved by transfer of momentum between objects.
- Find out whether collisions are elastic or not by calculation.
- Use conservation of momentum concepts to solve exam-type problems.

### Key Concepts

When we start to take a look at a situation – we focus on certain objects, called the system. In most of the examples we will be using, the system is made of two or three objects.

Objects can collide or explode at a point and we need a way to describe the motion of these objects before and after.

### Momentum

The momentum of an object is the amount of motion it has. It is calculated by multiplying the object's mass (kg) and its velocity ( $\text{m}\cdot\text{s}^{-1}$ ).

$$p = mv$$

The answer is measured in  $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$  and is a **vector**. This means that direction plays an important role in an object's momentum.

### Impulse

When objects with momentum touch:

- They exert forces on one another.
- Newton's third law states that the force between them is exactly the same, but opposite in direction.
- They touch each other (and exert the same force on one another) for the same amount of time.
- This will cause an equal but opposite change in their momenta, conserving the total momentum in the system.

Impulse is the amount of change in motion an object has because it experiences a force for some time.

$$\Delta p = F\Delta t$$

Note: the force used in this equation is a **VECTOR**. This means that force can be positive or negative to show its direction.

One of the ways to think about impulse is that it is the way that objects trade or swap momentum with one another.

### Conservation of Momentum

As mentioned before, impulse is the way that objects trade momentum on one another, allowing the total momentum of the system to remain constant.

When approaching a question of this type, it is important to note that **direction plays a key role**.

The idea is as follows:

- The total momentum of all objects is added up before the collision
- The total momentum of all the object is added up after the collision
- These two totals are made equal and any unknown values are found.

There are some important steps in approaching these questions. We will use a simple example later to show you how they work.

#### Step 1: Diagram and Direction.

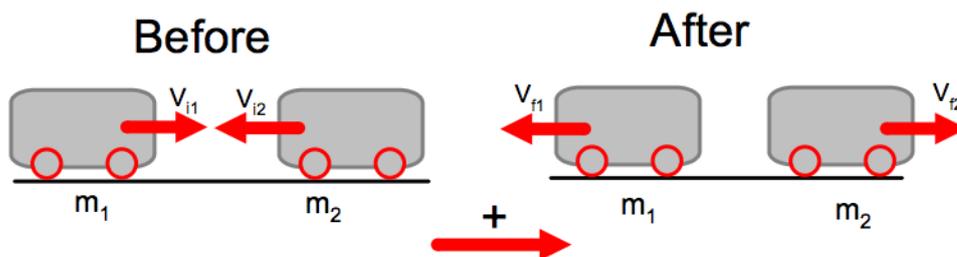
As with any good vector question, diagram and direction is the first step. Quickly draw a sketch and indicate which direction will be your positive direction. It may be chosen by the question but **stick to it!** Don't ever change the answers of your calculations to suit your personal feelings or change positive directions during the question.

You will need to have a diagram for the objects **before** and **after**.

#### Step 2: Conservation Equation.

Now, we are ready to set up our conservation of momentum equation. How we do this depends on how many objects there were before and after the collision.

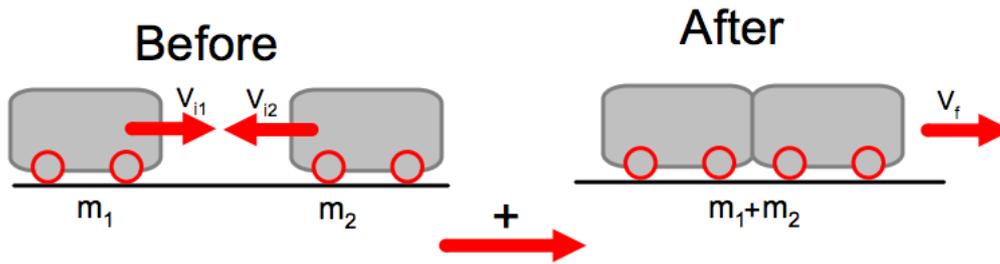
Note: each object needs its own "mv" term in the equation, see below for ideas.



$$p_i = p_f$$

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

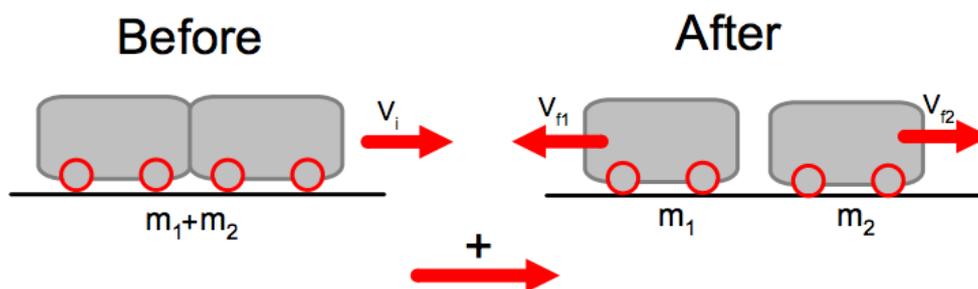
Notice how each separate object has its own "mv". Here, two objects collided and left separately so there were two objects before and two objects after. So the final equation has four pieces.



$$p_i = p_f$$

$$m_1 v_{i1} + m_2 v_{i2} = (m_1 + m_2) v_f$$

Here, two objects collided and combined so there were two objects before and one object after. So the final equation has three pieces.



$$p_i = p_f$$

$$(m_1 + m_2) v_f = m_1 v_{f1} + m_2 v_{f2}$$

Here, one object exploded and separated so there was one object before and two objects after. So the final equation has three pieces.

### Elastic Collisions

We know that momentum is conserved in a collision but what about the other quantity of motion, kinetic energy?

When a collision conserves kinetic energy we say that the collision is **elastic**. If kinetic energy afterwards does not equal the amount that there was before, the collision was **inelastic**. Often we have to prove this using calculations of  $K = \frac{1}{2}mv^2$  for all objects before and after.

Example:

**DOE March 2011 Question 4**

Two shopping trolleys, X and Y, are both moving to the right along the same straight line. The mass of trolley Y is 12 kg and its kinetic energy is 37,5 J.

- 1) Calculate the speed of trolley Y.

This is a simple energy calculation:

$$K = \frac{1}{2}mv^2$$

$$37,5 = \frac{1}{2}(12)v^2$$

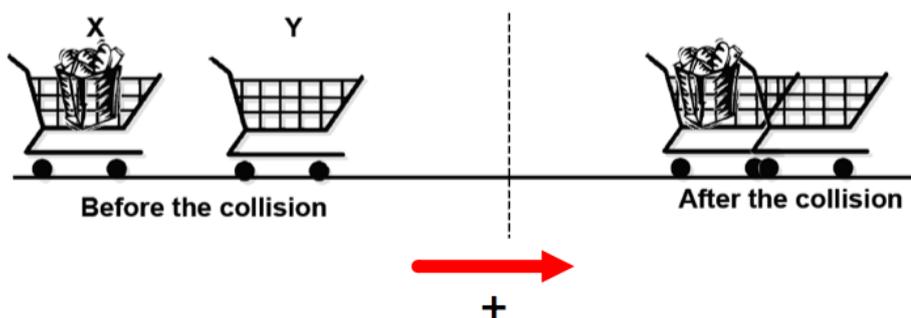
$$v = 2,5 \text{ m}\cdot\text{s}^{-1}$$

Now comes the momentum question:

Trolley X of mass 30 kg collides with trolley Y and they stick together on impact. After the collision, the combined speed of the trolleys is  $3,2 \text{ m}\cdot\text{s}^{-1}$ . (Ignore the effects of friction.)

- 2) Calculate the speed of trolley X before the collision.

**Step 1:** Diagram and Direction



Notice how the direction that we consider positive is marked with an arrow and a “plus” sign? This is the original direction of Trolley X.

**Step 2:** Conservation of momentum calculation

$$p_i = p_f$$

$$m_1v_{i1} + m_2v_{i2} = (m_1 + m_2)v_f$$

$$(30)v_{i1} + (12)(2,5) = (30 + 12)(3,2)$$

$$v_{i1} = 3,48 \text{ m}\cdot\text{s}^{-1} \text{ in original direction of motion.}$$

During the collision, trolley X exerts a force on trolley Y. The collision time is 0,2 s.

- 3) Calculate the magnitude of the force that trolley X exerts on trolley Y.

Here, we have an indication that time is playing a role. The only equation using time and momentum is:

$$F\Delta t = mv_f - mv_i$$

$$F(0,2) = (12)(3,2) - (12)(2,5)$$

$$F = 42 \text{ N in original direction of motion}$$

## Terminology

**System:** the collection of objects in question.

**Momentum:** the amount of motion a body has due to its mass and velocity.

**Conservation of Momentum:** the total linear momentum of an isolated will remain the same.

**Impulse:** the change in a object's momentum due to a force being exerted on it for a time.

**Collision:** the rapid striking of two or more objects together.

**Explosion:** the sudden, forceful separation of objects.

**Elastic collision:** collision where kinetic energy is conserved.

**Inelastic collision:** collision where kinetic energy is lost.

## Demonstration

In demonstrating the conservation of momentum – picture two rugby players running at one another. A lighter smaller player has to run at a faster speed to stop a larger player running in the opposite direction.

Remember why their combined momentum before and after is “0”. Their direction is opposite so the signs will be positive and negative – adding up to zero. If they stop one another – the sum is still “0” – showing that momentum was conserved.

## Questions

### Question 1:

(DOE March 2010 Question 3.2)

A net force  $F$  acts on each of two isolated objects, P and Q, as shown below.

The mass of Q is three times that of P. (Ignore the effects of friction.)



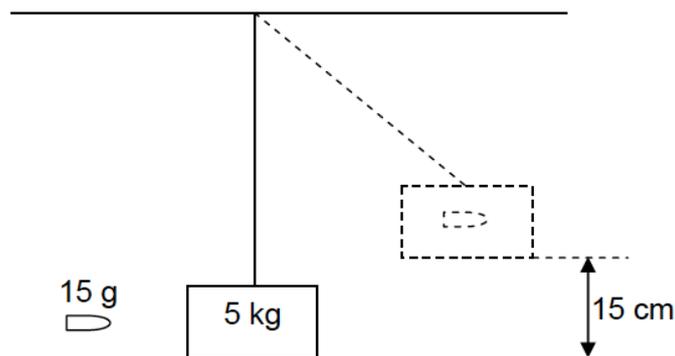
If the rate of change of momentum of object Q is  $x$ , then the rate of change of momentum of object P is as follows:

- A  $\frac{1}{9}x$
- B  $\frac{1}{3}x$
- C  $x$
- D  $3x$  (2)

### Question 2

(DOE Mar 2010 Question 4)

During an investigation a police officer fires a bullet of mass 15 g into a stationary wooden block, of mass 5 kg, suspended from a long, strong cord. The bullet remains stuck in the block and the block-bullet system swings to a height of 15 cm above the equilibrium position, as shown below. (Effects of friction and the mass of the cord may be ignored.)



- a.) State the law of conservation of momentum in words. (2)
- b.) Use energy principles to show that the magnitude of the velocity of the block bullet system is  $1,71 \text{ m}\cdot\text{s}^{-1}$  immediately after the bullet struck the block. (3)
- c.) Calculate the magnitude of the velocity of the bullet just before it strikes the block. (4)
- d.) The police officer is pushed slightly backwards by the butt of the rifle, which he is holding against his shoulder, whilst firing the rifle. Use the relevant law of motion to explain why this happens. (3)

## Links

- <http://www.mindset.co.za/learn/xtra>
- <http://www.education.gov.za/Examinations/PastExamPapers/tabid/351/Default.aspx>
- <http://phet.colorado.edu/en/simulation/collision-lab>