

SESSION 9: MEASUREMENT IN 2D AND 3D

Key Concepts

In this session we will focus on summarising what you need to know about:

- Measurements in 2D and 3D
- Perimeter and area
- Surface area and volume

X-planation

1. INTRODUCTION

Perimeter is always 1 dimension and the units are: mm, cm or m

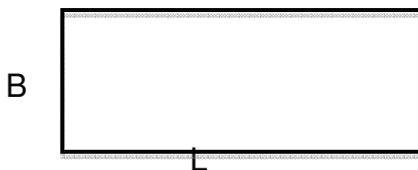
Area is always 2 dimensional and the units are mm^2 , cm^2 or m^2

Volume is always 3 dimensional and the units are mm^3 , cm^3 or m^3

2. PERIMETER AND AREA

a) Rectangle

- Perimeter of a rectangle = $2(L + B)$ (in sketch below)
- Area of rectangle = length x breadth = $L \times B$ (in sketch below).

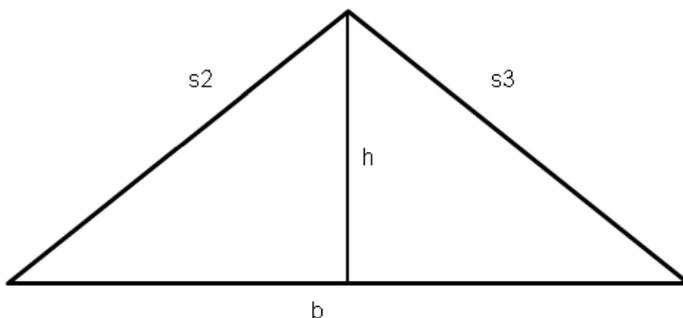


Example: If $L = 8\text{m}$ and $b = 4\text{m}$

Perimeter = $2(L + B)$	Area = LB
$P = 2(8 + 4)$	$A = 8 \times 4$
$P = 24\text{m}$	$A = 32\text{m}^2$

b) Triangle

- Perimeter of a triangle = all three sides added = $b + s_2 + s_3$ (in sketch below)
- Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{perpendicular height} = \frac{1}{2} \times b \times h$

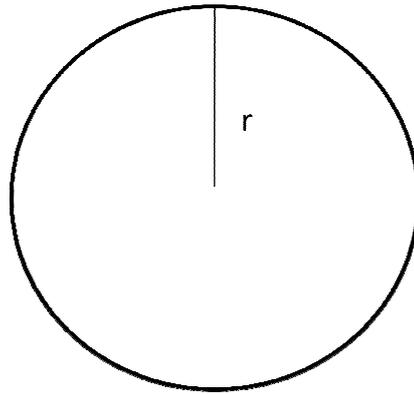


Example: If $b = 6\text{m}$; $h = 4\text{m}$; $s_2 = 5\text{m}$ and $s_3 = 5\text{m}$

$P = s_1 + s_2 + s_3$	$A = \frac{1}{2} \times b \times h$
$P = 6\text{m} + 5\text{m} + 5\text{m}$	$A = \frac{1}{2} \times 6 \times 4$
$P = 16\text{m}$	$A = 12\text{m}^2$

c) Circle

- Perimeter of a circle (circumference) = $2 \times \pi \times \text{radius} = 2\pi r$
- Area of a circle = $\pi \times \text{radius squared} = \pi r^2$
- Radius of a circle is the diameter $\div 2$



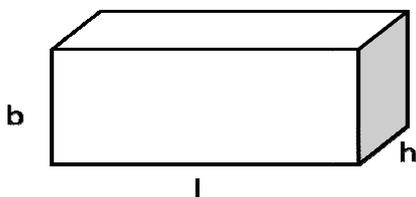
Example: If $r = 5\text{m}$

$P = 2\pi r$	$A = \pi r^2$
$P = 2 \times \pi \times 5\text{m}$	$A = \pi \times 5^2\text{m}$
$P = 31,42\text{m}$	$A = 78,4\text{m}^2$

3. SURFACE AREA AND VOLUME

a) Rectangular prism

- Surface area = $2(l \times b + l \times h + b \times h)$
- Volume = $l \times b \times h$

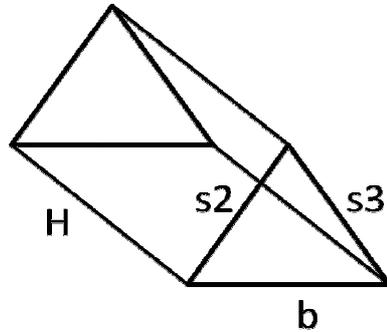


Example: If $l = 4\text{m}$; $b = 2\text{m}$ and $h = 3\text{m}$

$SA = 2(l \times b + l \times h + b \times h)$	$V = l \times b \times h$
$SA = 2(4\text{m} \times 2\text{m} + 4\text{m} \times 3\text{m} + 2\text{m} \times 3\text{m})$	$V = 4\text{m} \times 2\text{m} \times 3\text{m}$
$SA = 52\text{m}^2$	$V = 24\text{m}^3$

b) Triangular prism

- Surface area = $2\left(\frac{1}{2} \times b \times h\right) + s_1 \times H + s_2 \times H + s_3 \times H$
- Volume = $\frac{1}{2} \times b \times h \times \text{height of prism (H)}$



Example: If $b = 6\text{m}$; $h = 4\text{m}$; $s_2 = 5\text{m}$; $s_3 = 5\text{m}$ and $H = 3\text{m}$

$$SA = 2\left(\frac{1}{2} \times b \times h\right) + s_1 \times H + s_2 \times H + s_3 \times H$$

$$SA = (6\text{m} \times 4\text{m} + 6\text{m} \times 3\text{m} + 5\text{m} \times 3\text{m} + 5\text{m} \times 3\text{m})$$

$$SA = 72\text{m}^2$$

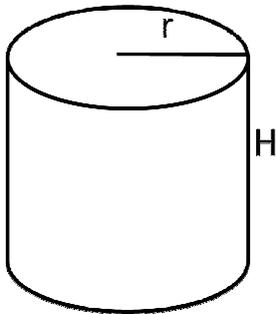
$$V = \frac{1}{2} \times b \times h \times H$$

$$V = \frac{1}{2} \times 6\text{m} \times 4\text{m} \times 3\text{m}$$

$$V = 36\text{m}^3$$

c) Cylinder

- Surface area = $2 \times \pi \times r^2 + 2 \times \pi \times r \times H$
- Volume = $\pi \times r^2 \times H$



Example: If $r = 3\text{m}$ and $H = 4\text{m}$

$$SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times H$$

$$SA = 2 \times \pi \times 3^2\text{m} + 2\text{m} \times \pi \times 3\text{m} \times 4\text{m}$$

$$SA = 131,95\text{m}^2$$

$$V = \pi \times r^2 \times H$$

$$V = \pi \times 3^2\text{m} \times 4\text{m}$$

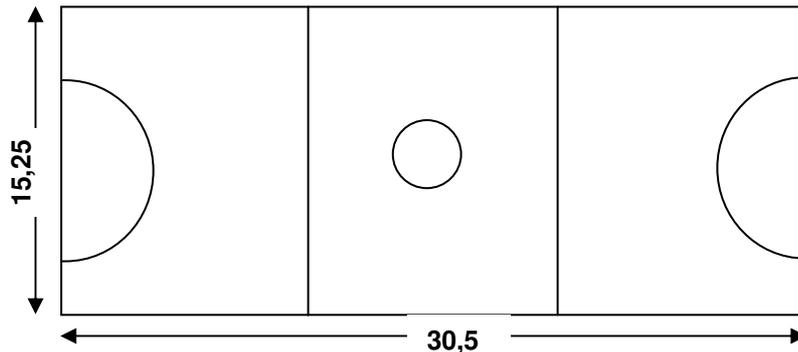
$$V = 113,1\text{m}^3$$

X-ample Questions

Question 1

Netball is a game played between two teams with seven players on each team. The rectangular netball court is divided into three equal sections with a centre starting circle with radius of 0,45m and two semi-circles at each end marking out the goal-shooting areas with radius 4,9m, as shown in the layout plan below.

The measurements on the diagram (not drawn to scale) are given in metres.



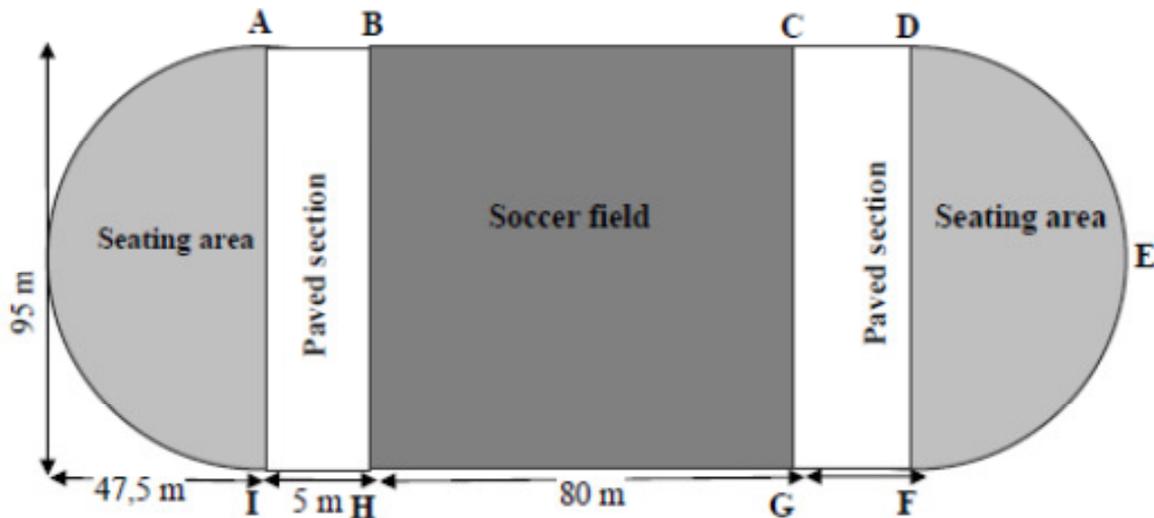
Your High School decides to build a grass netball court at their school and contracts Netball Incorporated to build the court.

- Calculate the area of the netball court in m^2 . (2)
- The cost per goal net is R124,80. Calculate the cost of TWO goal nets. (2)
- Lines will be painted on the netball court using grass paint. Calculate length of the lines that need to be painted in m. (5)
- If it takes two workers one hour to paint the lines on the netball court, how many workers would be needed to complete the task in half the time? (2)

Question 2

Mr and Mrs Makona would like to make a donation to help upgrade the local sports field. The sketch below shows a plan of the field. The measurements on the diagram (not drawn to scale) are in metres.

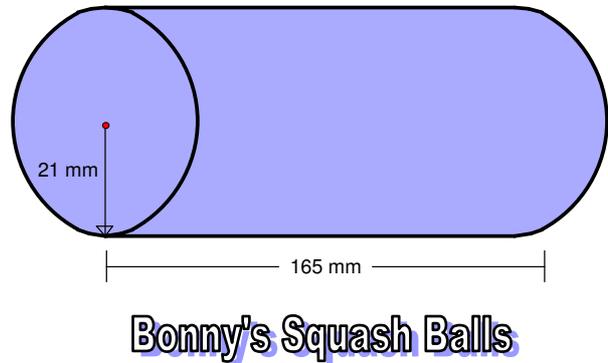
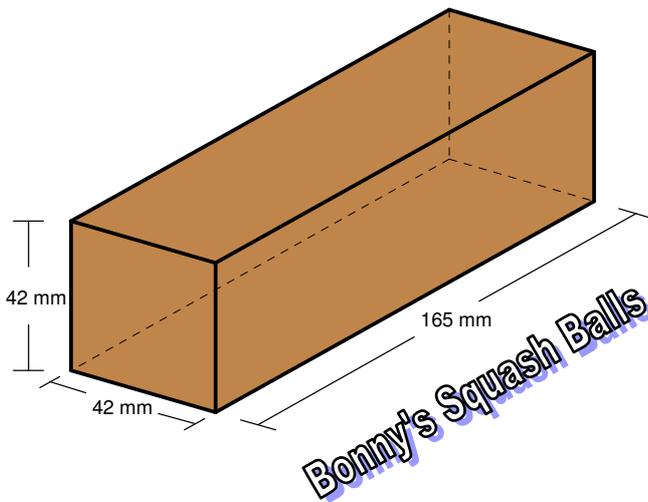
The sports field is made up of a rectangular grass soccer field ($80\text{m} \times 95\text{m}$), two semi-circular seating areas (radius 47,5m) and two rectangular paved areas (width 5m). The paved areas separate the seating areas from the soccer field.



- a) The Makonas donate R 3 000 to buy fertiliser for the soccer field.
- Calculate the area of the soccer field. (2)
 - Calculate how many bags of fertiliser can be bought with this donation if one bag of fertiliser costs R49. (3)
- b) The Makonas would also like to donate money to fence the perimeter of the sports field.
- Determine the length from A to D, the distance between the seating areas. (1)
 - Determine the length of the curved part of the seating area (DEF). (2)
 - Determine the total perimeter of the sports field. (2)
- c) If the total perimeter of the sports field is 478,46m and four gates, each 2m wide, are put in the fence, calculate how much fencing will be needed. (2)

Question 3

Neo is employed at a business that designs and makes packaging for small balls, fruit juice and biscuits. One of the first tasks that Neo is given, is to design packaging for squash balls. The pictures below show the dimensions of a rectangular box and a cylindrical container that Neo is considering using to package the squash balls.

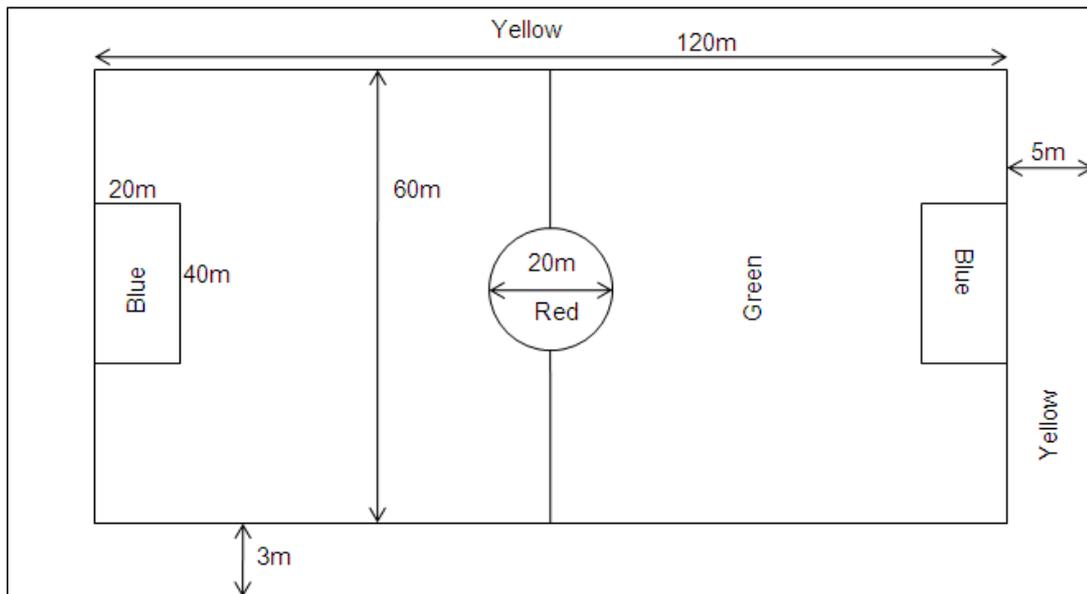


- a) If a squash ball has a diameter of 40 mm, how many squash balls is a box or a container designed to hold? (2)
- b) A net is a 2-dimensional picture of what an object looks like before it is folded into 3-dimensional shape. Draw a *rough* net for the rectangular box and for the cylindrical container.
 - The nets do not have to be drawn to scale.
 - The dimensions of the various parts of the box and container must be filled in on the net.
 (4)
- c) Calculate how much cardboard Neo needs to build the rectangular box. (2)
- d) Calculate how much cardboard Neo needs to build the cylindrical container. (3)
- e) Based on your calculations above, would it be cheaper for Neo to package the squash balls in the cylindrical container or the rectangular box? Explain your answer. (2)
- f) If you walk around the shops, you will notice that most squash ball manufacturers package their squash balls in rectangular boxes. Most golf ball and table tennis ball manufacturers also use rectangular boxes. Why do you think this is the case? (2)

X-ercise

After the huge success of the World Cup Soccer Tournament, the committee at the local soccer grounds wants to continue with the soccer fever. They investigate the possibility of creating a new design for their soccer field. They want to use colour-dyed grass in different areas of the field as shown in the accompanying diagram.

Area	Colour of Grass	Dimensions	Cost
Penalty area ($\times 2$)	Blue	Length = 40m Breadth = 20m	R125 per m^2
Centre circle	Red	Diameter = 20m	R120 per m^2
Rest of playing field	Green	Length = 120m Breadth = 60m	R75 per m^2
Border around the playing field	Yellow	Along the side = 3m Behind the post = 5m	R120 per m^2



- a) Calculate the area (in m^2) of the:
- Blue grass needed. (2)
 - Red grass needed. (2)
 - Green grass needed. (4)
- b) Calculate the cost of the red grass needed. (2)
- c) Thick, solid lines on the diagram indicate the markings needed in the game. These are drawn using special paint for sports fields. What length (in meters) of white paint is needed to mark the playing field (excluding that around the yellow border)? Give your answer to the nearest metre. (5)
- If we use 250ml of paint for every metre of marking, how much paint will be needed? Give your answer in litres. (3)
 - The white paint is sold in 2,5 litre tins. How many tins will be used on one field? (2)