

## Co-Ordinate Geometry Circles and Tangents

### Key Concepts

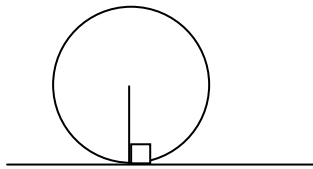
In this session we will focus on summarising what you need to know about:

- Derive a formula for circle centre the origin.
- Derive a formula for circle centre (a ; b).
- Define a tangent and use the formula
- Use Pythagoras
- Complete the square

### Terminology & definitions

Tangent: A line drawn with only one point of contact.

### Laws



Gradient of the radius is perpendicular to the gradient of the tangent

$$m_{\text{tangent}} \times m_{\text{radius}} = -1$$

### Equations

$$m_{\text{tangent}} \times m_{\text{radius}} = -1$$

$$x^2 + y^2 = r^2$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$y - y_1 = m(x - x_1)$$

### X-planation

- Always draw a sketch. This sketch need not be accurate
- Often the midpoint formula can be used if the diameter is involved
- To find the equation of a circle if the centre is the origin, we need only one point
- To find the equation of a circle if the centre is not the origin, we need one point and the centre

## X-ample Questions

### Question 1

The equation of a circle is given by the equation:

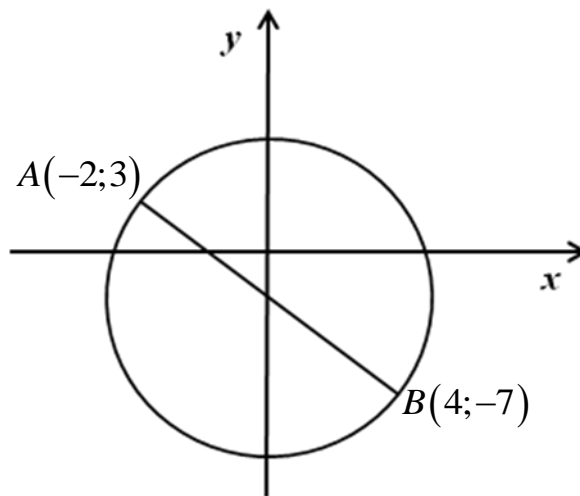
$$x^2 + y^2 - 6x - 3y = 8$$

Calculate:

- (a) The centre of the circle. (5)
- (b) The radius of the circle. (2)

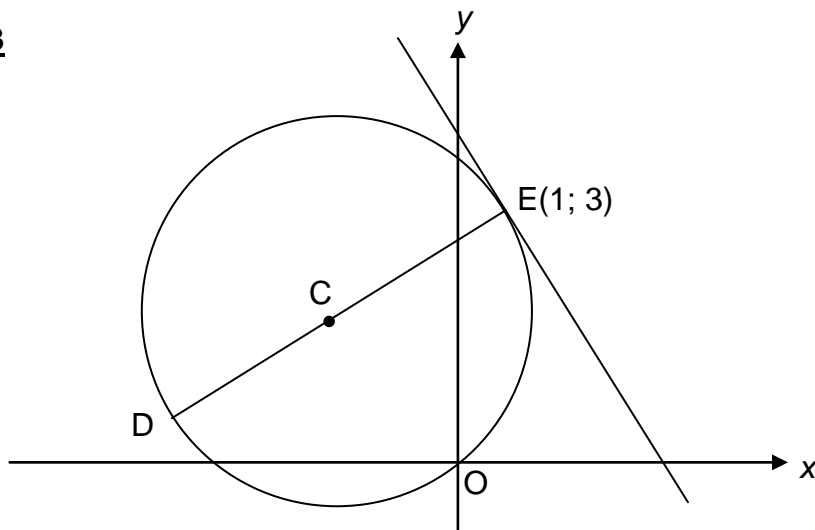
### Question 2

Refer to the diagram. Given a circle with diameter  $AB$  with  $A(-2;3)$  and  $B(4;-7)$ .



- (a) Determine the equation of the circle in the form  $(x-a)^2 + (y-b)^2 = r^2$ . (6)
- (b) Determine the equation of the tangent to the circle at A. (5)

### Question 3



In the figure, the circle with centre C is defined by the equation  $x^2 + y^2 + 3x - 4y - 1 = 0$  and the straight line DE is defined by  $5y - 2x - 13 = 0$ . E is the point (1; 3).

- (a) Determine the co-ordinates of the centre of the circle (4)
- (b) Calculate the length of the diameter of the circle, leaving the answer in simplified surd form, if necessary. (2)
- (c) Determine the equation of the tangent to the circle at E. (4)
- (d) Write down the coordinates of D (no lengthy calculations) (3)

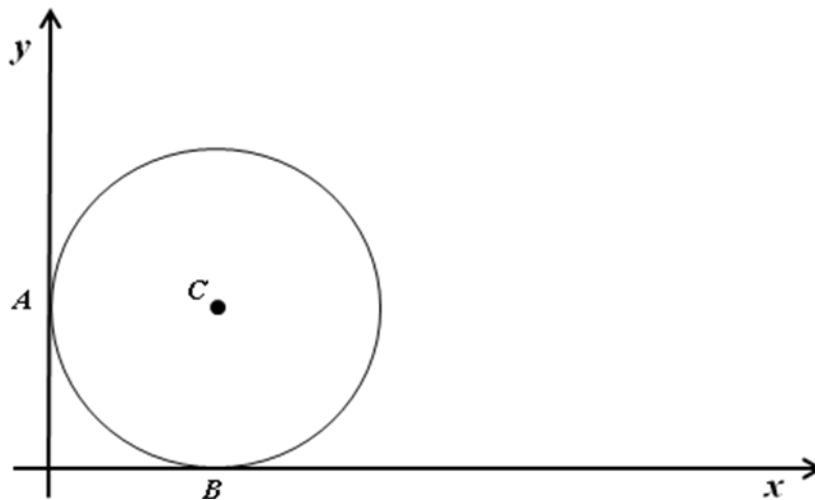
**Question 4**

The equation of a circle with radius  $5\sqrt{3}$  units is given by  $x^2 + y^2 - 10x + 8y - p = 0$ . Rewrite the equation in the form  $(x - a)^2 + (y - b)^2 = r^2$  and hence calculate the value of  $p$ .

**Question 5**

Refer to the diagram.

Given the circle's equation:  $(x - 2)^2 + (y - 2)^2 = 2^2$



- (a) Write down the coordinates of A, B and C. (3)
- (b) The circle is transformed according to the following rule:  $(x; y) \rightarrow (x + 2; y + 2)$   
Write down the coordinates of the centre and the length of the radius of the transformed circle. (3)
- (c) The original circle goes through an enlargement of factor  $k$  and produces a centre for the new circle of coordinates (6;6).
  - i) Write down the value of  $k$ . (1)
  - ii) Will the x- and y-axes still be tangents to the new circle? Motivate your answer. (2)
  - iii) Write down the equation of the new circle. (2)

## X-planation

### Transformations

- Translation  $(x; y) \rightarrow (x \pm a; y \pm b)$
- Reflection
  - About the y-axis  $(x; y) \rightarrow (-x; y)$
  - About the x-axis  $(x; y) \rightarrow (x; -y)$
  - About the line  $y=x$   $(x; y) \rightarrow (y; x)$
- Rotations
  - $90^0$  anti-clockwise  $(x; y) \rightarrow (-y; x)$
  - $270^0$  clockwise  $(x; y) \rightarrow (-y; x)$
  - $90^0$  clockwise  $(x; y) \rightarrow (y; -x)$
  - $270^0$  anti-clockwise  $(x; y) \rightarrow (y; -x)$
  - $180^0$  clockwise or anti-clockwise  $(x; y) \rightarrow (-x; y)$
- Enlargements
  - $(x; y) \rightarrow (kx; ky)$
  - If the scale factor is  $k$
  - Perimeter of new  $\rightarrow k$  Perimeter of original
  - Area of new  $\rightarrow k^2$  Area of original

### Question 6

A circle with centre  $T$  and equation  $x^2 + y^2 - 16x - 20y + 115 = 0$  and a circle with centre  $S$  and equation  $(x + 4)^2 + (y - 5)^2 = 36$  are given.

- 1) Use analytical methods to show that the circles touch each other. (6)
- 2) Determine the length of the tangent from the point  $P(8; -1)$  to the circle with centre  $S$ . (5)

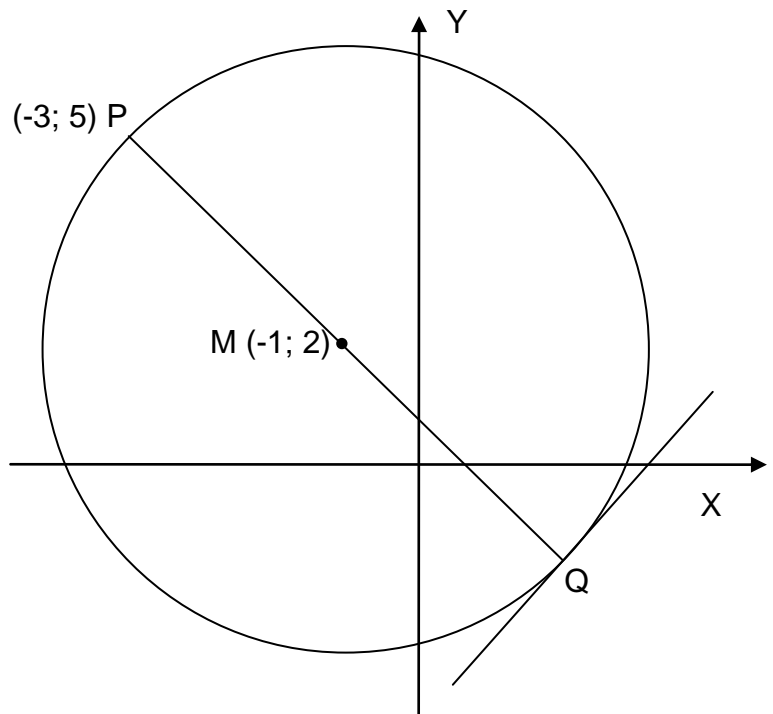
### Question 7

If the radius of the circle  $x^2 + y^2 - 4x + 6y = a$  is half the radius of the circle  $x^2 + y^2 - 4x + 6y = 3$ , calculate the numerical value of  $a$ . (6)

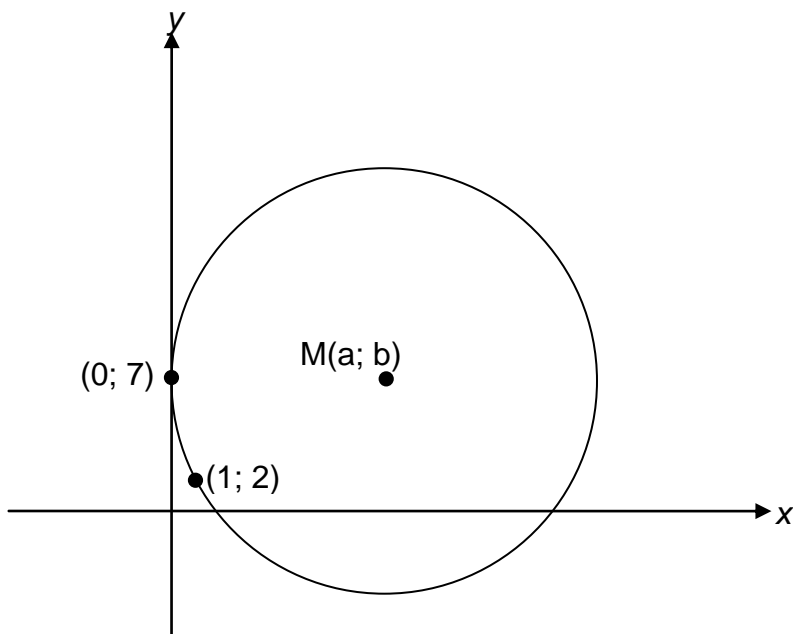
## X-ercise

### Question 1

PQ is the diameter of a circle, centre M (-1; 2). The co-ordinates of P are (-3; 5). Determine the co-ordinates of A, the point where the tangent drawn to the circle at Q, cuts the x-axis.



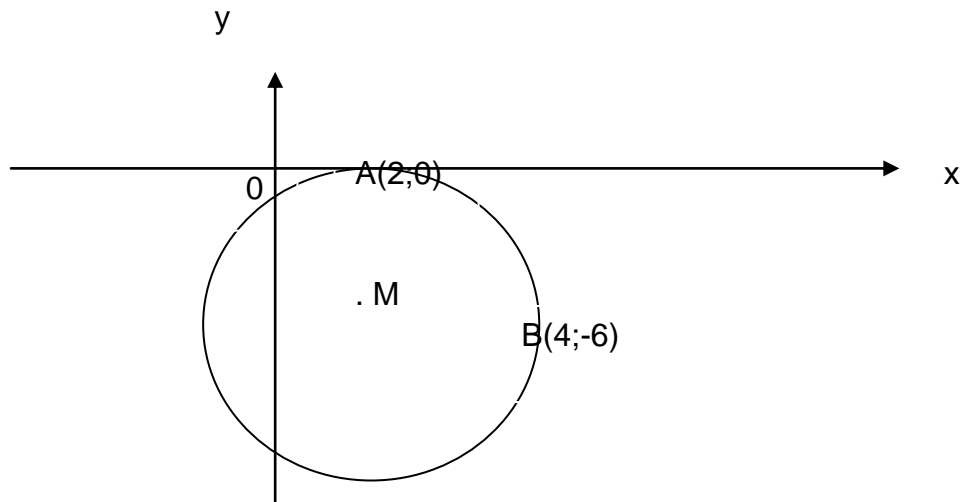
### Question 2



A circle with centre  $M(a; b)$  is tangential to the  $y$ -axis at  $(0; 7)$ . The circle also passes through the point  $(1; 2)$ .

- (a) Write down the value of  $b$ , with reason. (2)  
(b) Hence, write down an equation in ' $a$ ' and show that  $a = 13$ . (6)  
(c) Write down the equation of the circle. (2)

### Question 3



The circle touches the  $x$ -axis at  $A(2;0)$  and passes through  $B(4;-6)$ .

- (a) What is the value of  $x$  - value of the co-ordinate M? (1)  
(b) Hence, find the equation of the circle with centre M. (6)  
(c) If the co-ordinates of M are  $\left(2; -\frac{10}{3}\right)$ , find the equation of the tangent to the circle at B. (5)

### Answers

1  $\left(\frac{5}{2}; 0\right)$

2 (a)  $b = 7$  tangent (y-axis) perpendicular to radius.

2 (b)  $(a - 0)^2 + (7 - 7)^2 = (a - 1)^2 + (7 - 2)^2$

2 (c)  $(x - 13)^2 + (y - 7)^2 = 169$

3 (a)  $x = 2$

3 (b)  $(x - 2)^2 + \left(y + \frac{10}{3}\right)^2 = \frac{100}{9}$

3 (c)  $4y = 3x - 36$