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SESSION 12: TRIGONOMETRY: SINE, COSINE & AREA RULES

Key Concepts

In this session we will focus on summarising what you need to know about:

- Sine rule
- Cosine rule
- Area rules

X-planation

Solving two-dimensional problems using the sine, cosine and area rules

- The **sine-rule** can be used when the following is known in the triangle:

  - more than 1 angle and a side
  - 2 sides and an angle (not included)

  $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- The **cosine-rule** can be used when the following is known of the triangle:

  - 3 sides
  - 2 sides and an included angle

  $$a^2 = b^2 + c^2 - 2bc\cos A$$

- The **area** of any triangle can be found when at least two sides and an included angle are known.

  $$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin C$$

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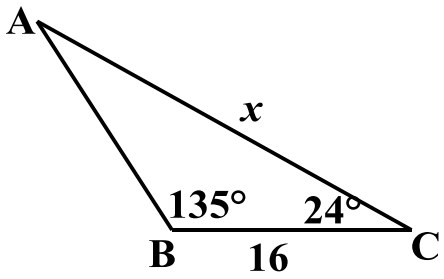
X-ample Questions

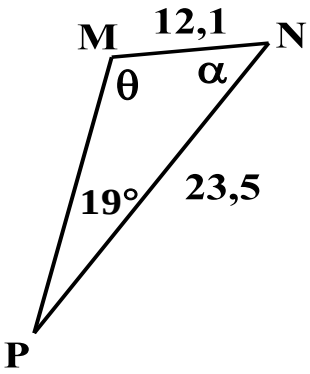
Question 1 (round off your answers to two decimal places)

(a) Determine x:



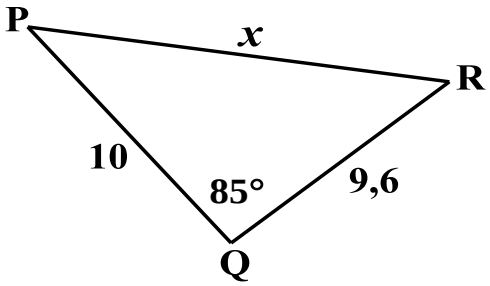
(4)

(b) Determine θ and α:



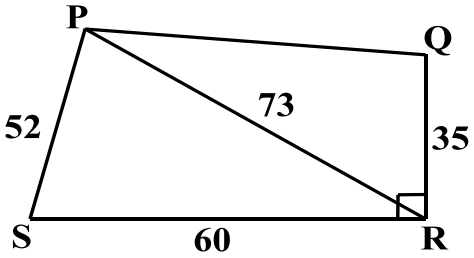
(4)

(c) Determine x: (3)



Question 2

Refer to the diagram and hence calculate:



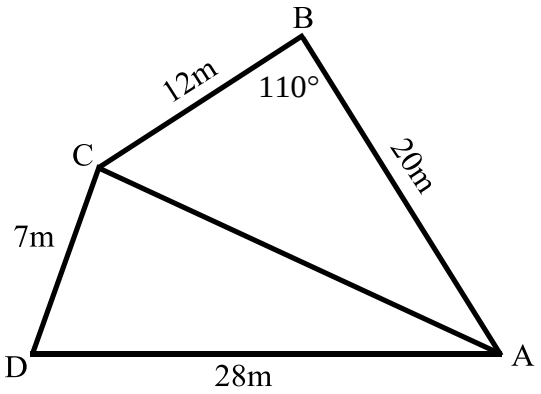
(a) $P\hat{R}S$ (4)

(b) Area of ΔPQR (4)

Question 3

A piece of land has the form of a quadrilateral $ABCD$ with $AB = 20\text{m}$, $BC = 12\text{m}$, $CD = 7\text{m}$ and $AD = 28\text{m}$. $\hat{B} = 110°$.

The owner decides to divide the land into two plots by erecting a fence from A to C.



(a) Calculate the length of the fence AC correct to one decimal place. (2)

(b) Calculate the size of $B\hat{A}C$ correct to the nearest degree. (2)

(c) Calculate the size of $\hat{D}$, correct to the nearest degree. (3)

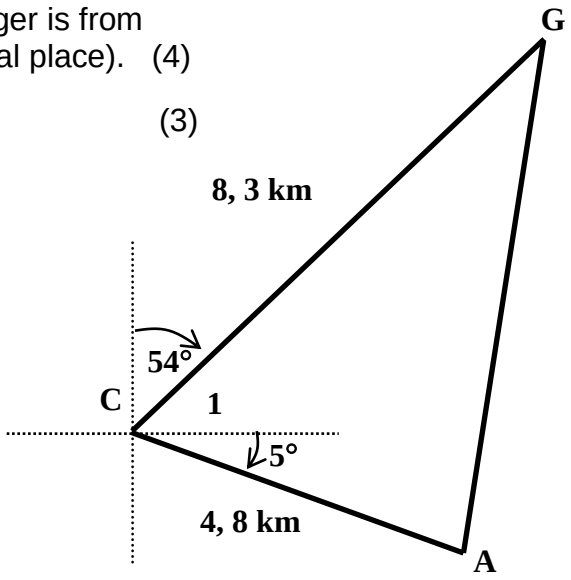
(d) Calculate the area of the entire piece of land ABCD, correct to one decimal place. (3)

Question 4

A game ranger G is 8,3 km from control centre C at a bearing of 54° east, when he receives a call that there is an injured antelope A that needs attention. The antelope is located 4,8 km at a bearing 5º south of east from the control centre. The diagram below is a representation of the above-mentioned situation.

(a) Calculate how far the game ranger is from the injured antelope (one decimal place). (4)

(b) Calculate the area of ΔGCA. (3)



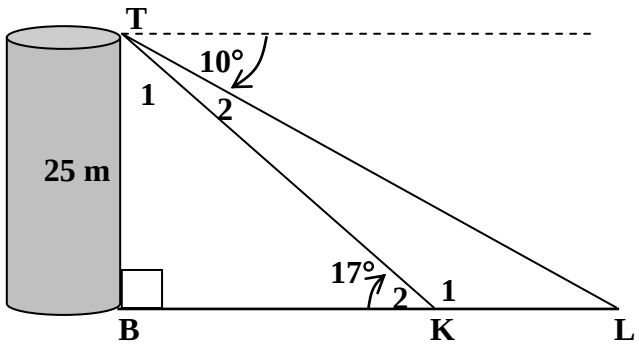
Question 5

The diagram below is a representation of a 25 m vertical observation tower TB, and two cars, K and L, on a road. The angle of depression from T to car L is 10°. The angle of elevation from car K to the top of the tower is 17°. B, K and L lie in a straight line and lie on the same horizontal plane as the base of the tower.

(a) Calculate the size of $\hat{L}$. (1)

(b) Calculate the length of KT. (3)

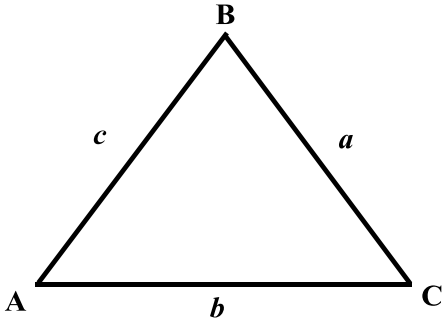
(c) Hence calculate the distance between the two cars. (4)



Question 6

ΔABC is an isosceles triangle with $AB = BC$ and $AB = c$, $AC = b$ and $BC = a$. By using the cosine rule, prove that

$$\cos B = 1 - \frac{b^2}{2a^2}$$ (4)



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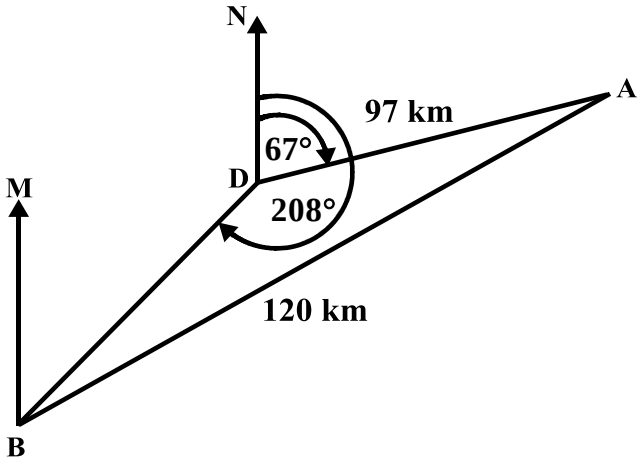
X-ercises

Question 1

Two ships, A and B, are 120 km apart. Ship A is at a bearing of $67°$ from D and 97 km away from D. DN points due north. Ship B is at a bearing of $208°$ from D.

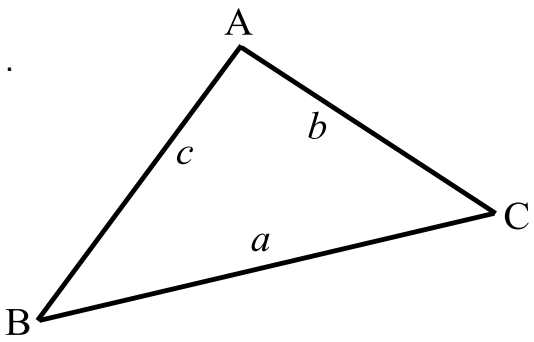


(a) Determine the bearing of Ship A from Ship B, that is, $M\hat{B}A$, when $BM||DN$. (6)

(b) If Ship B travels due north, and Ship B travels due south, then at some instant of time, Ship A is due east of Ship B. Calculate the distance between the two ships at that instant. (3)

Question 2

If $b = c$ and $a^2 = 7b^2$, show why it is impossible to construct ΔABC.



(4)