MATHEMATICS
Grade 12

LIVE: TRIG & ANALYTICAL GEOMETRY 02 JULY 2014

Lesson Description

In this lesson we:

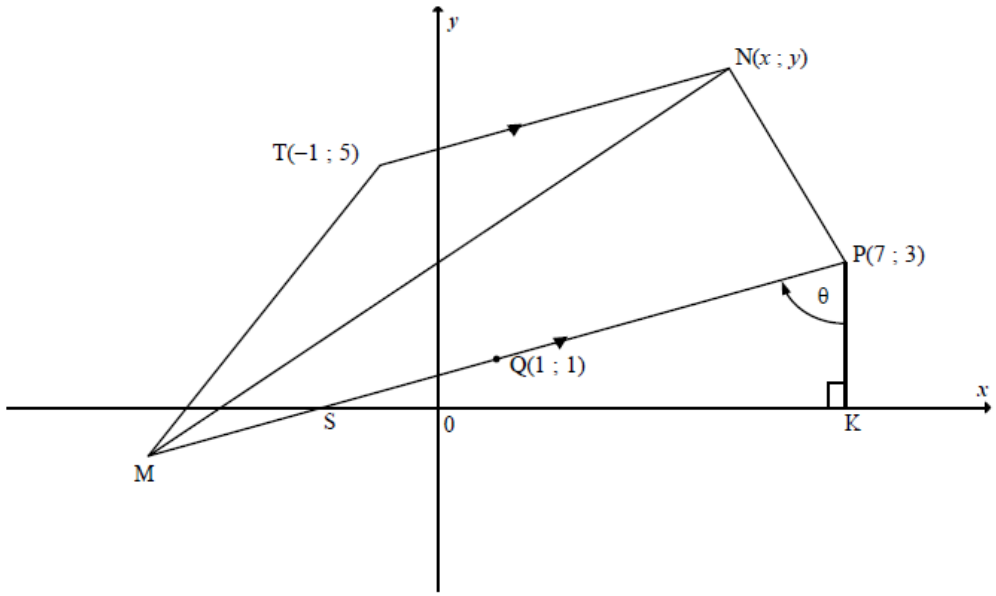
- Work through various examination questions relating to Trig and Analytical Geometry.

Improve your Skills

Question 1

(Adapted from Exemplar 2014, Paper 2, Question 3)

In the diagram below, M, T(–1 ; 5), N(x ; y) and P(7 ; 3) are vertices of trapezium MTNP having TN | | MP. Q(1 ; 1) is the midpoint of MP. PK is a vertical line and $K\hat{P}S = \theta$.
The equation of NP is y = –2x + 17.



1.1 Write down the coordinates of K. (1)

1.2 Determine the coordinates of M. (2)

1.3 Determine the gradient of PM. (2)

1.4 Calculate the size of θ. (3)

1.5 Hence, or otherwise, determine the length of PS. (3)

1.6 Determine the coordinates of N. (5)

1.7 If A(a ; 5) lies in the Cartesian plane:

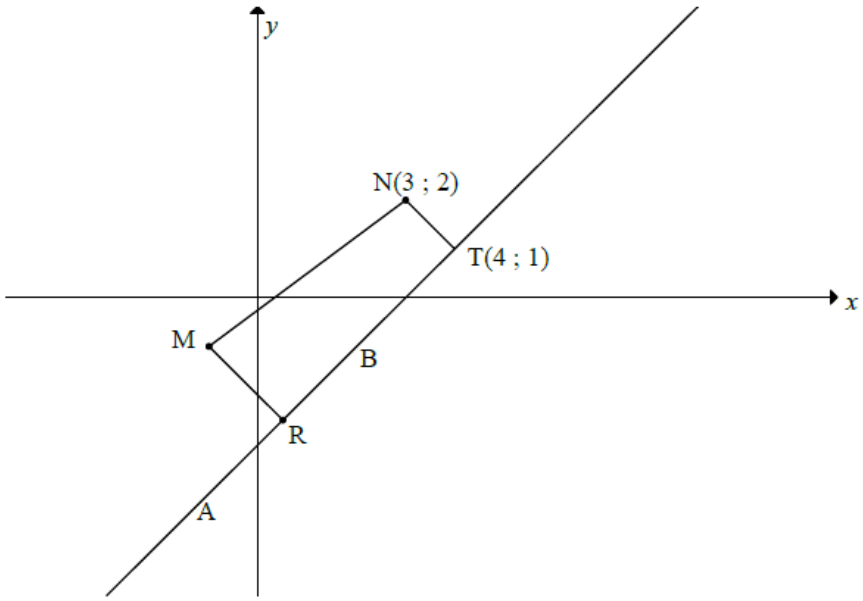
1.7.1 Write down the equation of the straight line representing the possible positions of A. (1)

1.7.2 Hence, or otherwise, calculate the value(s) of a for which $Q\hat{A}T = 45°$. (5)

Question 2

(Adapted from Exemplar 2014, Paper 2, Question 4)

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3 ; 2) at point T(4 ; 1).



2.1 Write down the coordinates of M. (1)

2.2 Determine the equation of AT in the form $y = mx + c$. (5)

2.3 If it is further given that MR = $\frac{\sqrt{10}}{2}$ units, calculate the length of AB.

Leave your answer in simplest surd form. (4)

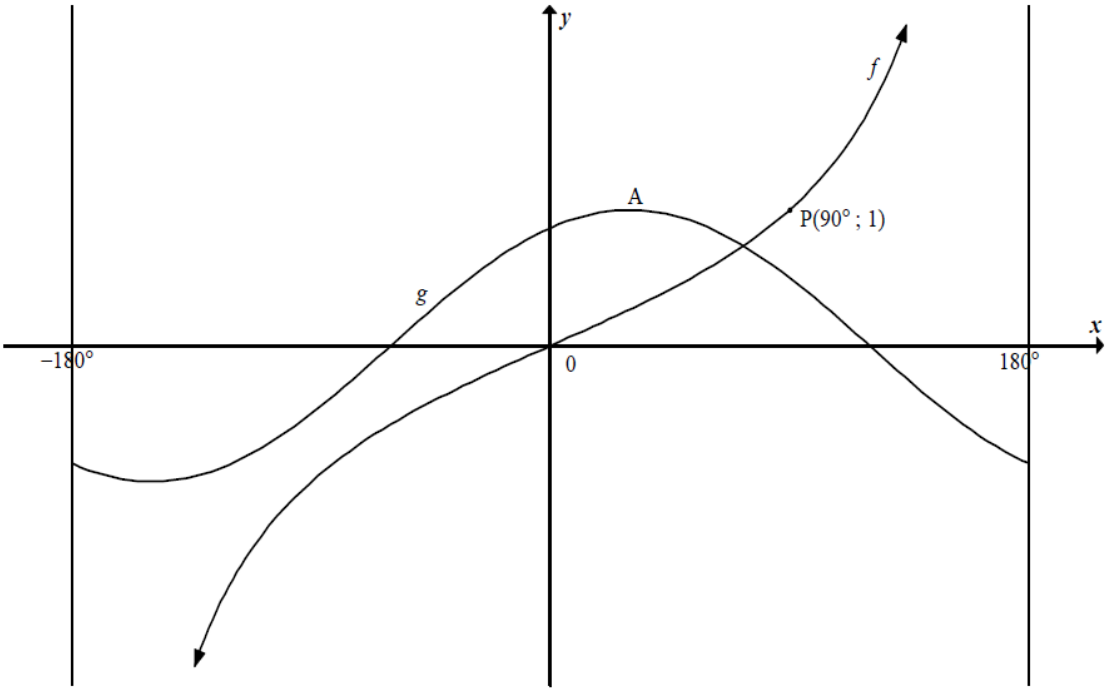
2.4 Calculate the length of MN. (2)

2.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3)

Question 3

(Adapted from Exemplar 2014, Paper 2, Question 6)

In the diagram below, the graphs of f (x) = tan bx and g(x) = cos (x – 30°) are drawn on the same system of axes for –180° ≤ x ≤ 180°. The point P(90° ; 1) lies on f. Use the diagram to answer the following questions.



3.1 Determine the value of b. (1)

3.2 Write down the coordinates of A, a turning point of g. (2)

2.3 Write down the equation of the asymptote(s) of y = tan b(x + 20°) for x ∈ [–180°; 180°]. (1)

2.4 Determine the range of h if h(x) = 2g(x) + 1. (2)

Question 4

(Adapted from Exemplar 2014, Paper 2, Question 5)

4.1 Given that $\sin\alpha = -\frac{4}{5}$ and 90° < α < 270°.

WITHOUT using a calculator, determine the value of each of the following in its simplest form:

4.1.1 sin (–α) (2)

4.1.2 cos α (2)

4.1.3 sin (α – 45°) (3)

4.2 Consider the identity: $\dfrac{8\sin(180°-x)\cos(x-360°)}{\sin^2 x-\sin^2(90°+x)} = -4\tan 2x$

4.2.1 Prove the identity. (6)

4.2.2 For which value(s) of x in the interval 0° < x < 180° will the identity be undefined? (2)

4.3 Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)