SINE, COSINE AND AREA RULES

18 AUGUST 2014

Lesson Description

In this lesson we:

- Learn the tools which will assist in solving non-right angled triangles
- Learn the conditions under which to apply the Sine and Cos Rule.
- Use the Sine Rule:

  In any $\triangle ABC$: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ or $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

- Use the Cosine Rule:

  In any $\triangle ABC$:
  $$a^2 = b^2 + c^2 - 2bc \cos A$$
  $$b^2 = a^2 + c^2 - 2ac \cos B$$
  $$c^2 = a^2 + b^2 - 2ab \cos C$$

- Use the Area Rule

$$Area\ \Delta ABC = \frac{1}{2}ab\sin C$$
$$Area\ \Delta ABC = \frac{1}{2}ac\sin B$$
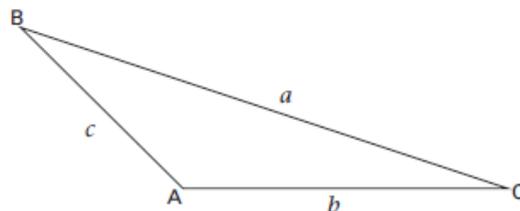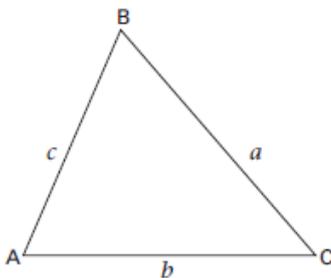$$Area\ \Delta ABC = \frac{1}{2}bc\sin A$$

Summary

Cosine Rule

We will apply the cosine rule when we are given:

- The length of two sides of a triangle and the size of the included angle (SAS)
- The length of all three sides of the triangle (SSS)

The cosine rule

In any $\triangle ABC$:
$$a^2 = b^2 + c^2 - 2bc \cos A$$
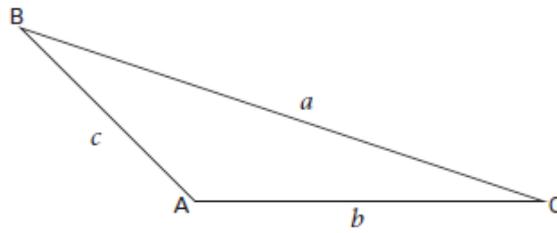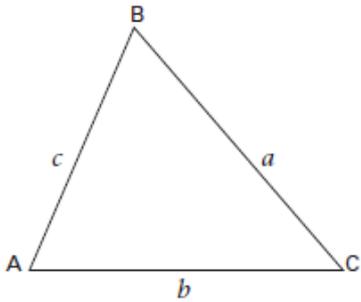$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Sine Rule

We will apply the sine rule when we are given:

- The length of one side and the size of two angles of a triangle (SAA)
- The length of two sides and the size of a non-included angle (SSA).

The sine rule

In any $\triangle ABC$: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ or $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Use $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ when you want to calculate the size of an angle.

Use $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ when you want to calculate the length of a side.
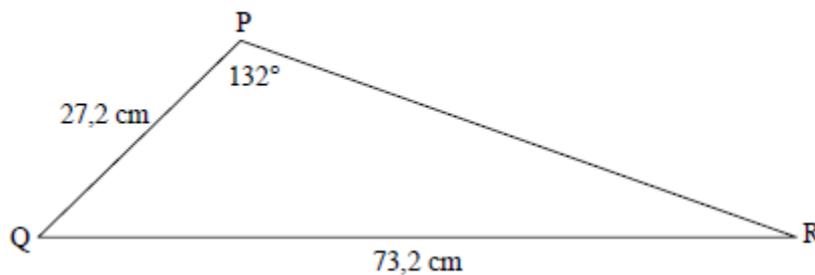
When using the sine rule, you need to have at least one angle **and** the side opposite that given angle.
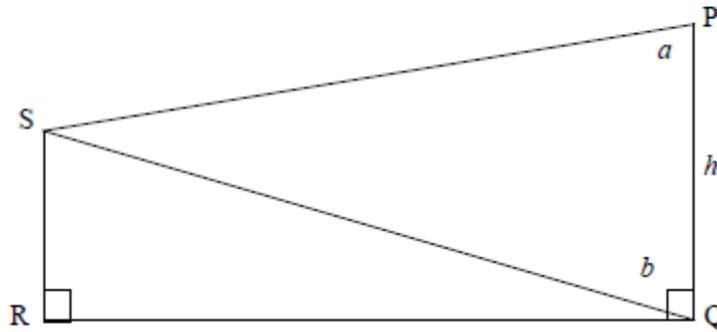
Test Yourself

Question 1

In $\triangle PQR$, $\hat{P} = 132°$, PQ = 27,2 cm and QR = 73,2 cm.

1.1 Calculate the size of $\hat{R}$. (3)

1.2 Calculate the area of $\triangle PQR$ (3)

Question 2

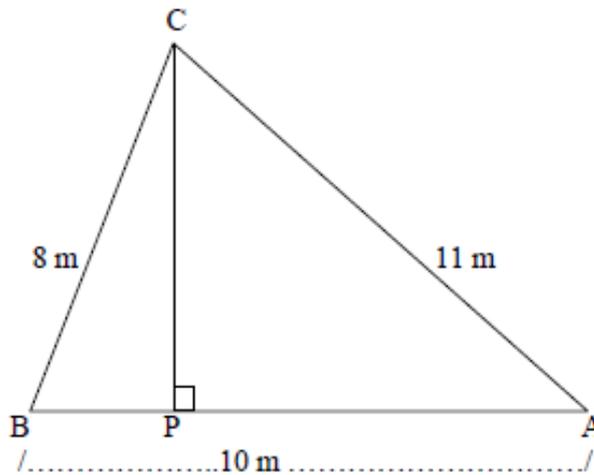In the figure below, $S\hat{P}Q = a$, $P\hat{Q}S = b$ and PQ = h. PQ and SR are perpendicular to RQ.

2.1 Determine the distance SQ in terms of a, b and h. (3)

2.2 Hence show that RS = $\frac{h\sin a\cos b}{\sin(a+b)}$ (3)

Question 3

CP is a radio mast. CA and CB are cables used to support the mast. B and A are on the same level as P and $\triangle$CAB is formed. CA = 11 m, CB = 8 m and BA = 10 m.

3.1 Determine C$\hat{A}$B. (4)
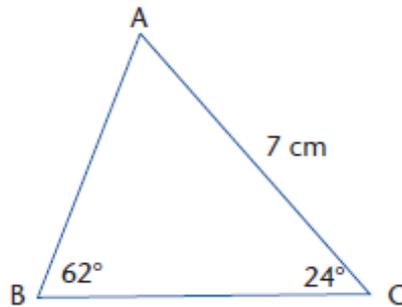
3.2 Hence, determine the height of the mast CP. (2)

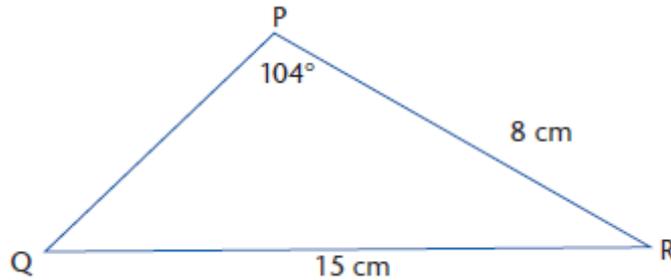3.3 Determine the area of $\triangle$CAB. (2)

Improve your Skills

Question 1
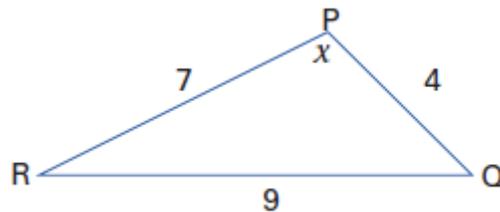
Calculate the length of AB.

Question 2

Calculate the size of $\hat{Q}$.

Question 3

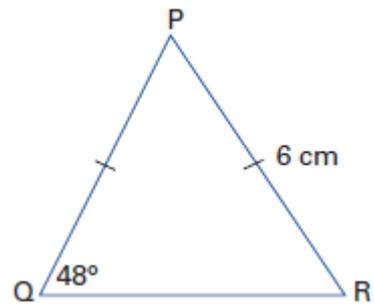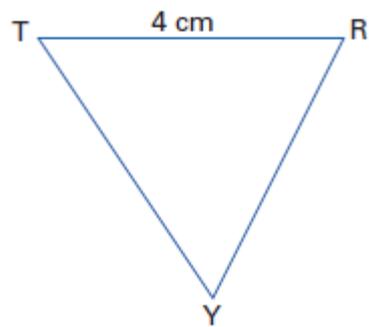Calculating the largest angle in a triangle with all three sides known:
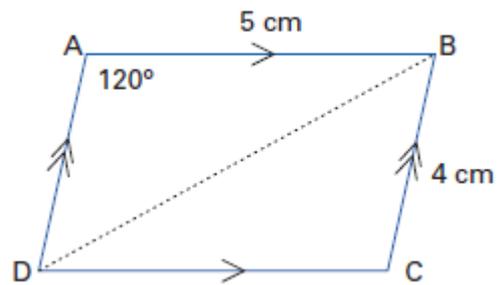
Question 4

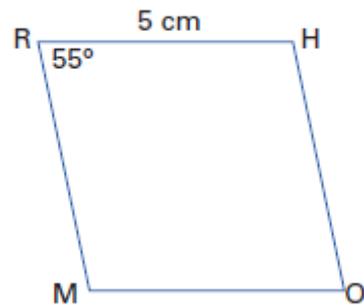Calculate the area of the following shapes:

a.) Isosceles triangle PQR, with PQ = PR

b.) Equilateral triangle TRY

c.) Parallelogram ABCD

d.) Rhomus RHOM

Question 5

TRAP is a trapezium with TR || PA.

    a.) Calculate the length of PR.
    b.) Calculate the size of $T\hat{R}P$
    c.) Hence determine the size of $R\hat{P}A$.
    d.) Calculate the area of TRAP.