

GEOMETRY OF CIRCLES: CYCLIC QUADRILATERALS & TANGENTS

4 AUGUST 2014



Lesson Description

In this lesson we:

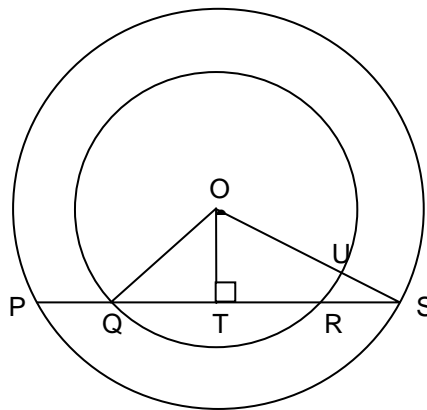
- Apply the theorems about cyclic quadrilaterals and tangents to a circle to solving riders



Challenge Question

Two concentric circles, centred at O , have radii of 5 cm and 8,5 cm respectively.

$QR = 6$ cm and $OT \perp PS$.



Determine the length of PS .



Summary

(Opp. \angle 's of cyclic quad.)

- A tangent to a circle is perpendicular to the radius at the point of contact.

(Tan \perp Rad)

- Two tangents drawn to a circle from the same point outside the circle are equal in length.

(Tan. from a common pt.)

- The exterior angle of a cyclic quad is equal to the interior opposite angle.

(Ext. \angle of cyclic quad.)

- The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.

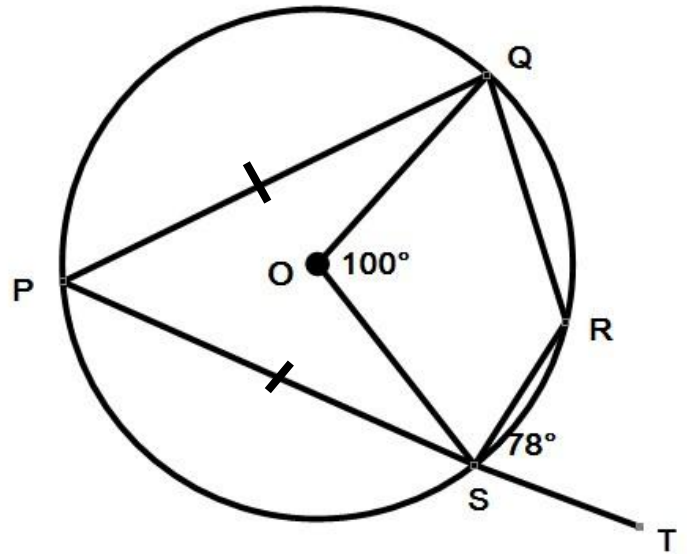
(Tan Chord Thm OR Alternate Seg Thm)



Test Yourself

Refer to the diagram alongside for
Questions 1 – 5

P, Q, R and S are points on the circumference of a circle, centre, O. PST is a straight line. $PQ = PS$
Angle $SOQ = 100^\circ$ and angle $RST = 78^\circ$



Question 1

Find the size of angle QRS

- | | | | | |
|------------|------------|------------|------------|-------------|
| A. | B. | C. | D. | E. |
| 50° | 13° | 56° | 26° | 116° |

Question 2

Find the size of angle PQS

- | | | | | |
|---------------|---------------|---------------|---------------|----------------|
| A. 50° | B. 52° | C. 40° | D. 65° | E. 116° |
|---------------|---------------|---------------|---------------|----------------|

Question 3

Find the size of angle OQS

- | | | | | |
|---------------|---------------|---------------|---------------|----------------|
| A. 50° | B. 52° | C. 40° | D. 26° | E. 116° |
|---------------|---------------|---------------|---------------|----------------|

Question 4

Find the size of angle PSO

- | | | | | |
|---------------|---------------|---------------|---------------|----------------|
| A. 50° | B. 25° | C. 56° | D. 26° | E. 116° |
|---------------|---------------|---------------|---------------|----------------|

Question 5

Find the size of angle SQR

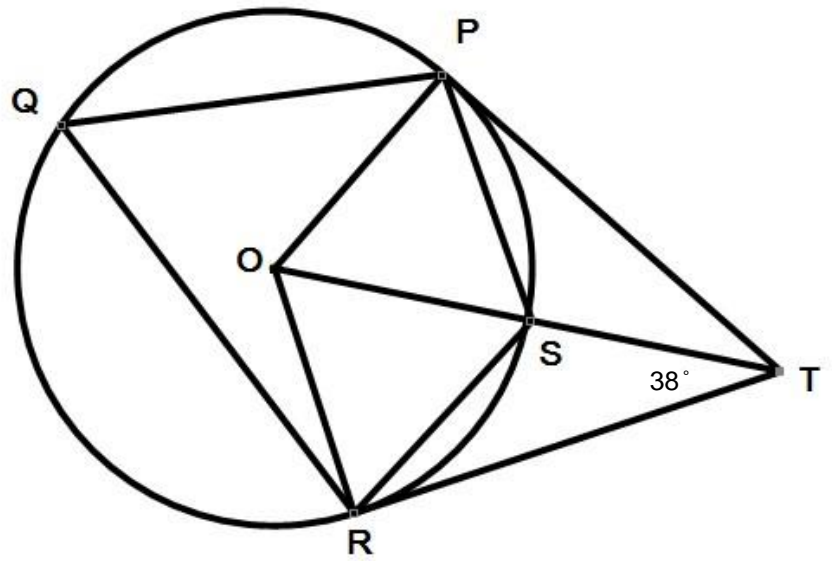
- | | | | | |
|---------------|---------------|---------------|---------------|----------------|
| A. 13° | B. 52° | C. 13° | D. 26° | E. 116° |
|---------------|---------------|---------------|---------------|----------------|

Refer to the diagram below for **Questions 6 - 10**

P, Q, R and S are points on the circumference of a circle, centre, O. PT and TR are tangents to the circle. OST is a straight line.

Angle OTR = 38° .

:



Question 6

Find the size of the angle ROT

- A. 50° B. 52° C. 56° D. 64° E. 116°

Question 7

Find the size of the angle PQR

- A. 50° B. 52° C. 56° D. 64° E. 116°

Question 8

Find the size of the angle SRT

- A. 50° B. 52° C. 56° D. 26° E. 116°

Question 9

Find the size of the angle PSO

- A. 50° B. 52° C. 64° D. 26° E. 116°

Question 10

Find the size of the angle PST

- A. 50° B. 52° C. 56° D. 26° E. 116°



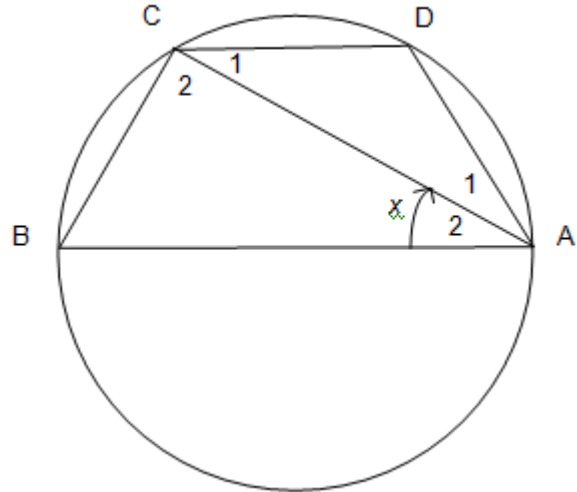
Improve your Skills

Question 1

In the diagram, AB is the diameter of circle ABCD.

$$\hat{A}_2 = x.$$

Express the size of \hat{D} in terms of x .



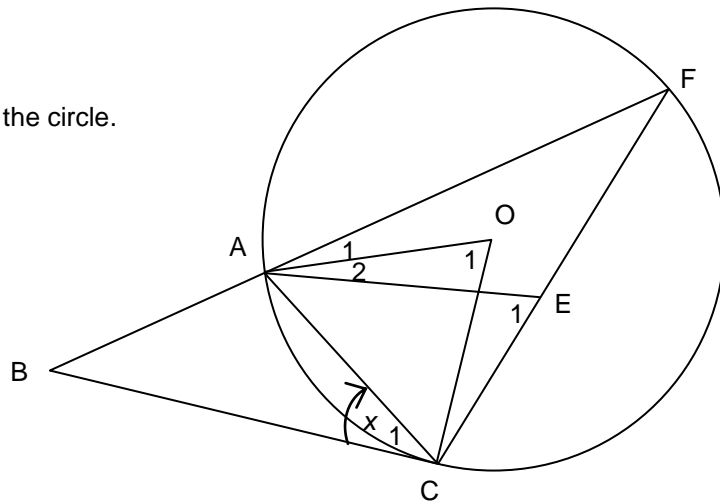
Question 2

O is the centre of the circle.

AE = EF and BC is a tangent to the circle.

BAF and CEF are straight lines.

$$\text{Let } \hat{C}_1 = x.$$



2.1 Express \hat{E}_1 in terms of x .

2.2 Express \hat{O}_1 in terms of x .

2.3 Why is OACE a cyclic quadrilateral?

notes for...

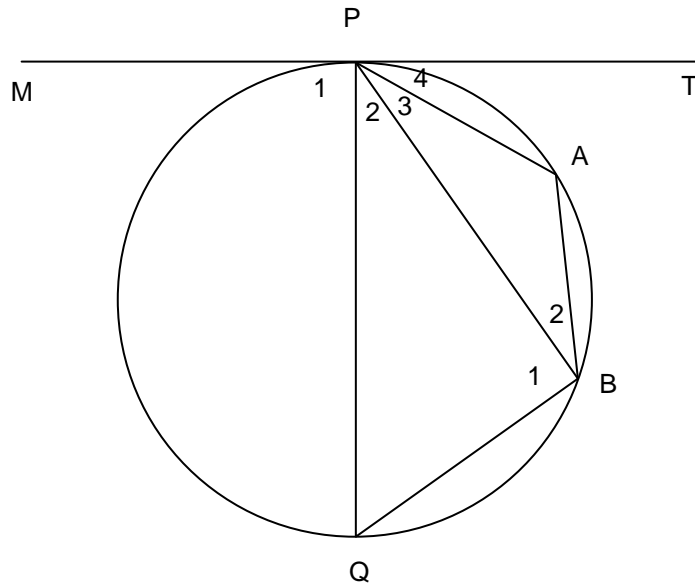
Question 3

PQ is a diameter. $\widehat{MPB} = 122^\circ$ and MT is a tangent.

Find the magnitude of:

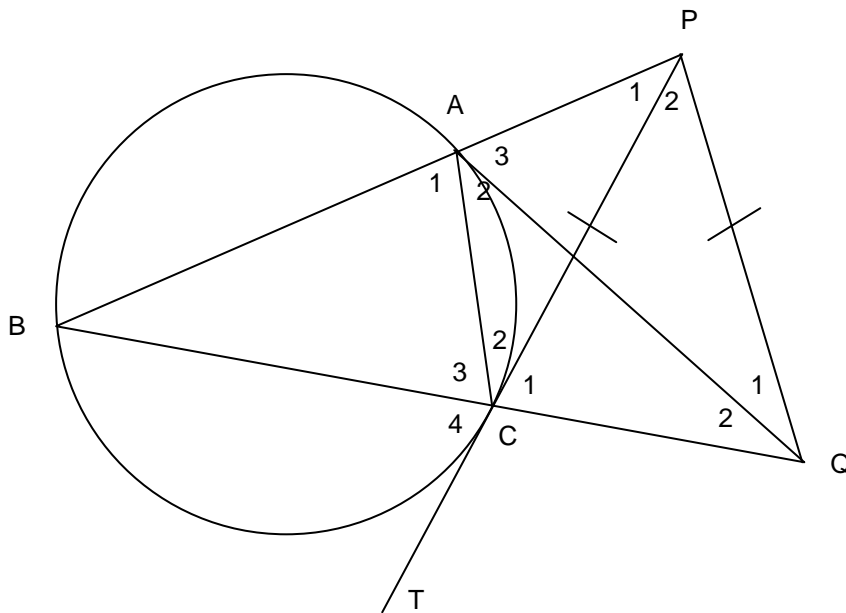
3.1 \widehat{Q}

3.2 \widehat{A}



Question 4

In the figure, $\triangle ABC$ is inscribed in a circle. $PQ = PC$. PC is a tangent at C to the circle through A, B and C.



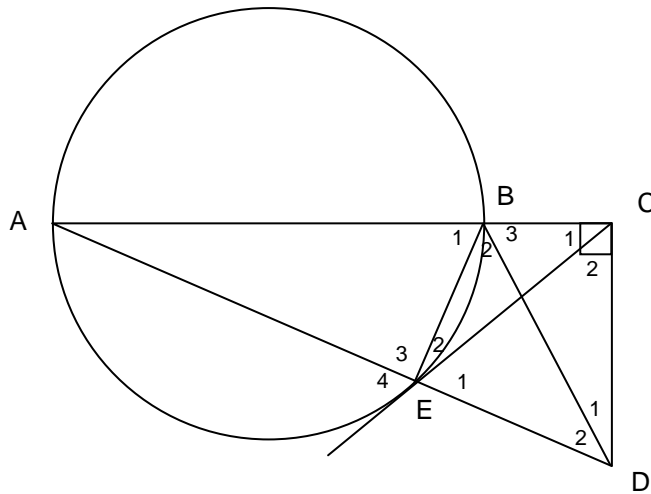
Prove that:

ACQP is a cyclic quadrilateral.

(6)

Question 5

In the diagram below, diameter AB is produced to C. CE is a tangent to the circle at E. AE is produced to D and $DC \perp AC$.



Prove the following:

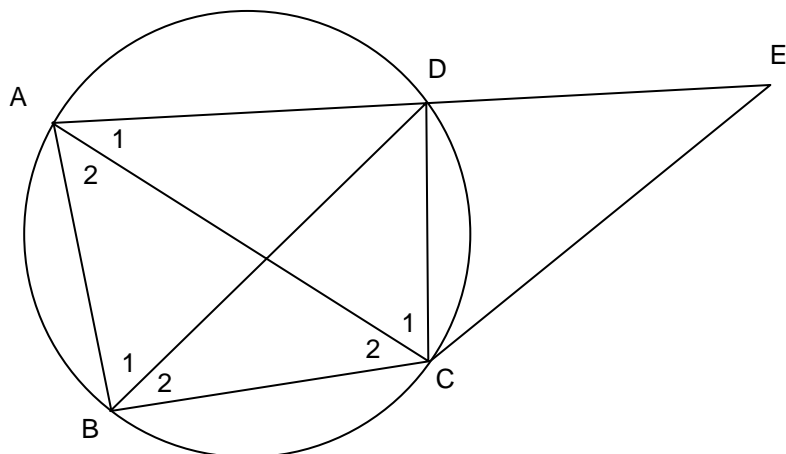
- 5.1 BEDC is a cyclic quadrilateral
- 5.2 $\hat{D}_1 = \hat{A}$
- 5.3 $CE = CD$
- 5.4 $\hat{B}_1 = \hat{B}_3$

Question 6

ABCD is a cyclic quadrilateral with AD produced to E such that $DE = DC$.

C and E are joined and DB is the angle bisector of $\hat{A}BC$.

Let $\hat{B}_1 = x$.



- 6.1 Prove that $\hat{C}_1 = \hat{B}_2$.
- 6.2 Prove that $AD = DC$.
- 6.3 Find \hat{CDE} in terms of x .

Question 7

In the figure below P, A, R, C, Q and B are points on the circumference of the circle.

7.1 Complete: $\hat{P} + \hat{C}_2 = 180^\circ$ (.....)

7.2 Prove that $\hat{P} + \hat{Q} + \hat{R} = 360^\circ$

