

GEOMETRY OF CIRCLES: CHORDS & ANGLES

28 JULY 2014



Lesson Description

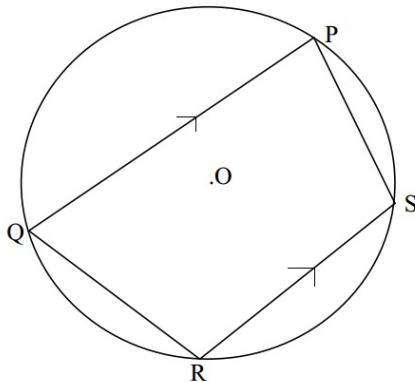
In this lesson we:

- Work through key concepts and questions relating to Euclidean Geometry.



Test Yourself

Question 1



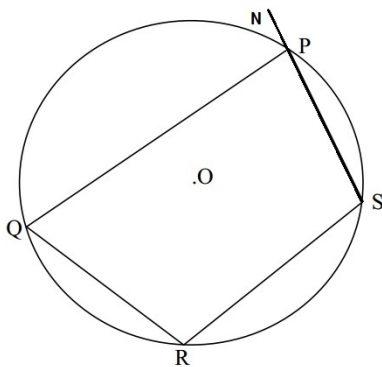
Angle $PQR = 68^\circ$ Select the correct options for the other angles

- A $R = S = 112^\circ$
- B $R = 68^\circ = S$
- C $P = 68^\circ$
- D $P = 112^\circ$

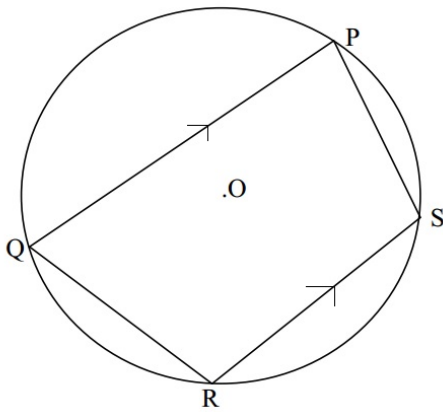
Question 2

Angle NPQ is equal to angle.....

- A PSR
- B SPQ
- C RQP
- D QRS



Question 3

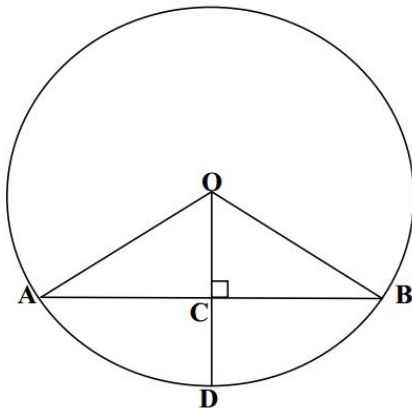


Angle $PQR = 68^\circ$

What reason would you give for saying that $QRS = 112^\circ$?

- A Exterior angle of cyclic quad
- B Interior angles of cyclic quad are supplementary
- C Co-interior angles between parallel lines are supplementary
- D Base angles of a trapezium

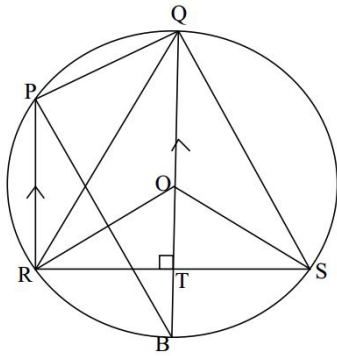
Question 4



Given that O is the centre of the circle and that $OB = 13$ units and that $AB = 10$ units. The length of OC is equal to

- A 12 units
- B 10 units
- C 8 units
- D 5 units

Question 5



Select all the angles which are equal to RPB?

- A PBQ
- B BQS
- C BQR
- D QRP



Summary

1. The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.
(Line from centre to midpt of chord)
2. The angle which an arc of a circle subtends at the centre of the circle is twice the angle it subtends at any point on the circumference.
(\angle at centre = $2 \times \angle$ at circumf)
3. The angle subtended at the circumference by a diameter is a right angle.
(\angle in semi-circle)
4. Angles in the same segment of a circle are equal. (Also known as the 'Butterfly Theorem'.)
(\angle 's in the same seg.)
5. The opposite angles of a cyclic quad are supplementary.
(Opp. \angle 's of cyclic quad.)
6. A tangent to a circle is perpendicular to the radius at the point of contact.
(Tan \perp Rad)
7. Two tangents drawn to a circle from the same point outside the circle are equal in length.
(Tan. from a common pt.)
8. The exterior angle of a cyclic quad is equal to the interior opposite angle.
(Ext. \angle of cyclic quad.)
9. The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.
(Tan Chord Thm OR Alternate Seg Thm)



Improve your Skills

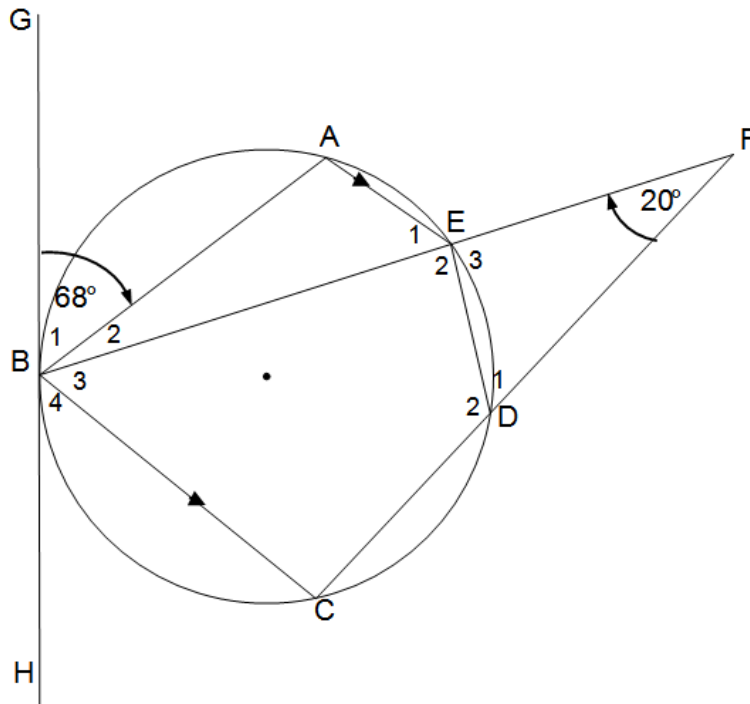
Question 1

a.) Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to ...

(1)

b.) In the diagram, A, B, C, D and E are points on the circumference of the circle such that $AE \parallel BC$. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68^\circ$ and $\hat{F} = 20^\circ$.

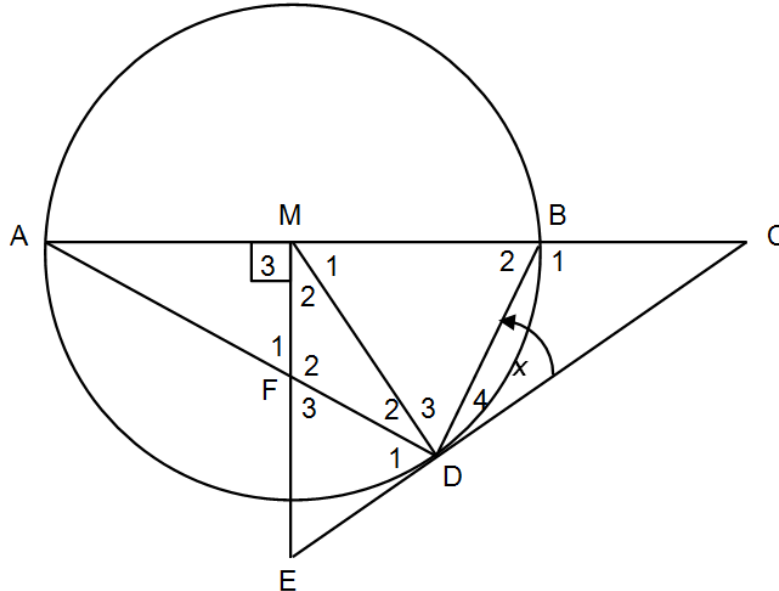


Determine the size of each of the following:

- | | | |
|------|-------------|-----|
| i. | \hat{E}_1 | (2) |
| ii. | \hat{B}_3 | (1) |
| iii. | \hat{D}_1 | (2) |
| iv. | \hat{E}_2 | (1) |
| v. | C | (2) |

Question 2

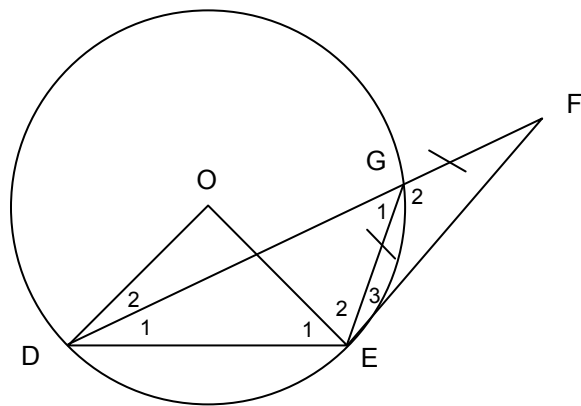
In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



- a) If $D_4 = x$, write down, with reasons, TWO other angles each equal to x . (3)
- b) Prove that CM is a tangent at M to the circle passing through M, E and D. (4)
- c) Prove that FMBD is a cyclic quadrilateral. (3)
- d) Prove that $DC^2 = 5BC^2$. (3)
- e) Prove that $\triangle DBC \sim \triangle DFM$. (4)
- f) Hence, determine the value of $\frac{DM}{FM}$. (2)

[19]

Question 3



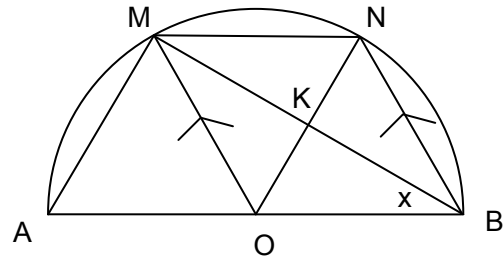
In the accompanying figure, FE is a tangent to the circle with centre O. D and F are joined so that EG = GF.

- a) If $\hat{E}_3 = x$, name, with reasons, two other angles each equal to x . (4)
- b) Prove that $DE = EF$. (2)
- c) Express $\hat{D\hat{O}E}$ in terms of x . (4)

[10]

Question 4

In the given figure, AOB is the diameter of the semi-circle, centre O , $MO \parallel NB$, ON and MB intersect at K and $\hat{M\hat{B}O} = x$.



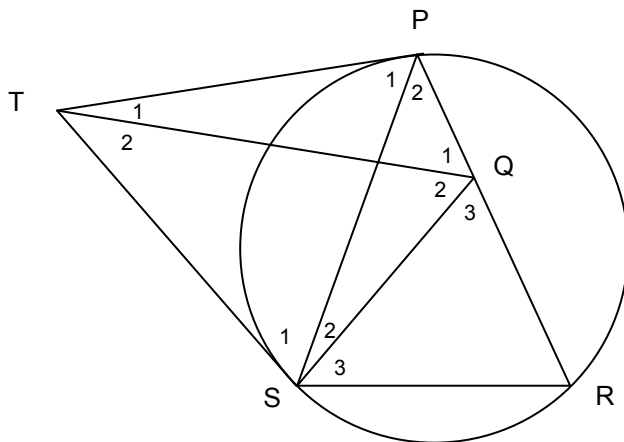
- a) Prove that MB bisects $\hat{N\hat{B}O}$. (5)
- b) Express $\hat{M\hat{K}N}$ in terms of x . (4)

It is further given that $x = 30^\circ$.

- c) Calculate the sizes of the angles of $\triangle MKN$. (4)
- d) Hence explain why $MOBN$ is a rhombus. (2)

[15]

Question 5



In the accompanying figure, TP and TS are tangents to the circle. R is a point on the circle and SR and PR are joined. Q is a point on PR so that $\hat{P}_1 = \hat{Q}_1$. S and Q are joined.

Prove that:

- a) $TQ \parallel SR$ (4)
- b) $QPTS$ is a cyclic quadrilateral. (4)
- c) TQ bisects $\hat{S\hat{Q}P}$. (4)

[12]