TRIGONOMETRY: REDUCTION FORMULAE 12 MAY 2014

Lesson Description

In this lesson we:

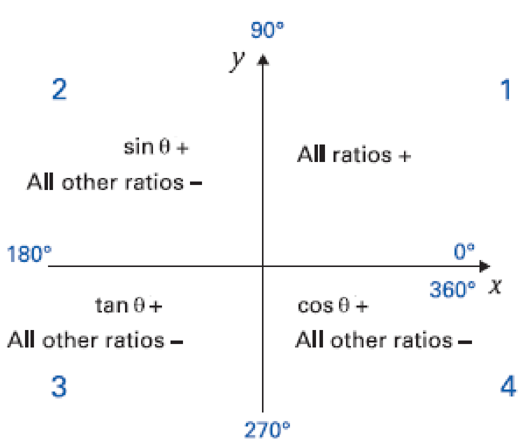
1. Work with the Reduction Formulae for the Trig Ratios of the following angles:
   - $180° - A$
   - $180° + A$
   - $360° - A$
   - $360° + A$
   - $90° - A$
   - $90° + A$
   - $-A$
2. Simplify numerical and algebraic trigonometric expressions

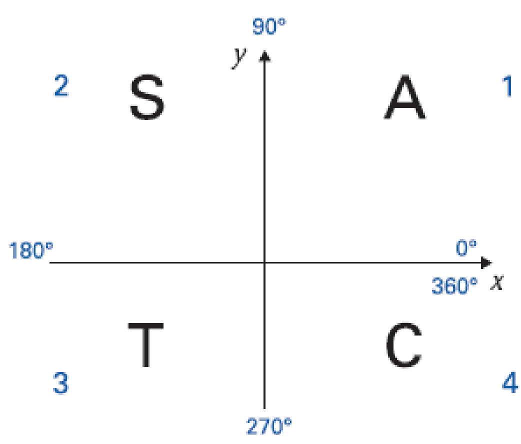
Summary

Definitions of the Ratios

Ratio	On a system of axes	In a right-angled triangle
$\sin\theta$	$\frac{y}{r}$	$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{h}$
$\cos\theta$	$\frac{x}{r}$	$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{h}$
$\tan\theta$	$\frac{y}{x}$	$\frac{\text{opposite}}{\text{adjacent}} = \frac{o}{a}$

Signs of the ratios in the Four Quadrants





Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

Stemming from the identity $\sin^2\theta + \cos^2\theta = 1$, we also get the following two:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Reduction formulae 180° ± θ, 360° - θ, 90° ± θ

- 180° and 360° lie on the x-axis. If one works of the x-axis with angles such as 180° – θ, 180° + θ, 360° – θ or – θ, the ratio is unchanged.
- When reducing ratios of angles ($90 \pm \theta$), the ratios change to the Co-functions.

Test Yourself

Question 1

$$\frac{\sin(A-180°)}{\cos(A+180°).\tan(-A)} = \cdots$$

A. $-tan^2A$
B. 1
C. tan^2A
D. 1

Question 2

If $tanx = a^{-1}$ with a > 0 and $x \in [0°; 360°]°$, then $sinx = \cdots$

A. $\frac{1}{\sqrt{1+a^2}}$
B. $\frac{-1}{1+a}$ or $\frac{1}{1+a^2}$
C. $\frac{1}{\sqrt{1+a^2}}$ or $\frac{-1}{\sqrt{1+a^2}}$
D. $1+a^2$

Question 3

If $cosx = \frac{-\sqrt{3}}{2}$ and $0° < x < 180°$, determine the value of $sinx$.

A. $\frac{-2}{\sqrt{3}}$
B. $\frac{1}{2}$
C. $-\frac{\sqrt{3}}{2}$
D. $-\frac{1}{2}$

Question 4

$\tan(180° - x).\tan(180° + x) = \cdots$

A. $-2tanx$
B. $-tan^2x$
C. $tan2x$
D. tan^2x

Question 5

$cos225° + 1 = \cdots$

A. $\frac{-2-\sqrt{2}}{2}$
B. $\frac{\sqrt{2}-2}{2}$
C. $\frac{2+\sqrt{2}}{2}$
D. $\frac{2-\sqrt{2}}{2}$

Question 6

$\frac{sin240°}{sin120°} = \cdots$

A. 1
B. $\frac{1}{2}$
C. -1
D. $-\frac{1}{2}$

Question 7

The function $y = tanbx$ is undefined for $x = \mp 30°$ *and* $x = \mp 90$, $x = [-90°; 90°]$. What is the value of *b*?

A. 1
B. $\frac{1}{2}$
C. 3
D. 2

Question 8

$\frac{\cos(180° - A)}{\sin(90° - A)} = \cdots$

A. -1
B. $-tanA$
C. 1
D. $\frac{1}{-tanA}$

Question 9

The sign of $sinA$ is the same as the sign of $cosA$, but opposite to the sign of $tanA$. Which statement is true?

A. $0° < A < 90°$
B. $90° < A < 180°$
C. $180° < A < 270°$
D. $270° < A < 360°$

Improve your Skills

Question 1

1.1 If $\cos 65° = m$, and $\cos\theta = m$, give FIVE possible values for θ

1.2 P is the point $(-1; b)$ and $X\hat{O}P = 210°$

a.) Use a diagram to calculate the value of *b*.
b.) If Q is another point on the terminal ray OP, write down the possible coordinates of Q.

Question 2

If $\sin 16° = K$, write each of the following in terms of K:

a.) $\sin 196°$
b.) $\cos 74°$
c.) $\sin(-344°)$
d.) $(1 - \cos^2 164°)$

Question 3

3.1 $$\frac{1}{4\tan^2(-30°)}+\tfrac{3}{2}\cos 240° - 2\sin 1395°$$

3.2 $$\frac{\cos(90°-x).\cos(1080°+x)}{\cos(180°-x).\cos(-x)\left[\tan(180°+x)-\dfrac{1}{\tan(360°-x)}\right]}$$