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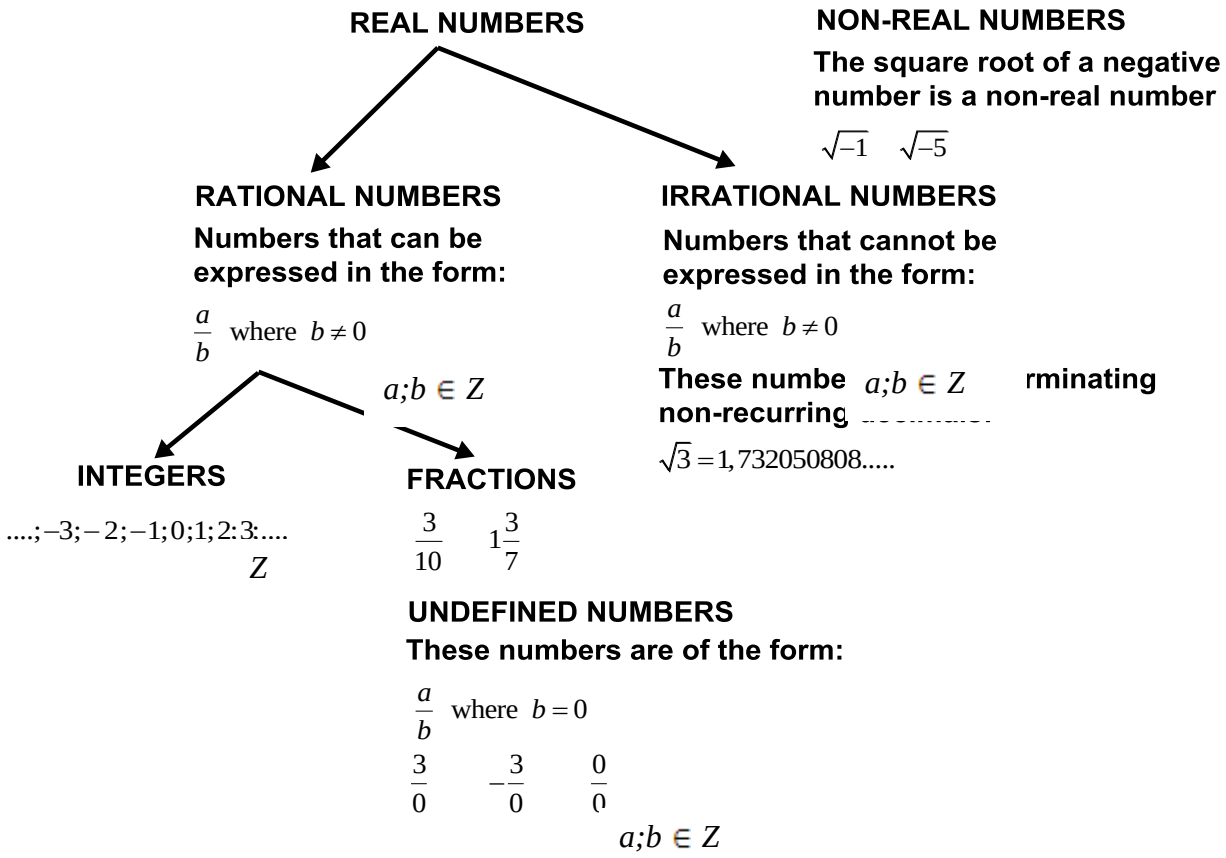
SESSION 1: REAL NUMBERS, EXPONENTS & SURDS

X-planation

Summary of the number system



Relationship of the number system to square roots

The square root of a number can either be real or non-real depending on the type of number under the square root.

Consider the expression: $\sqrt{x}$

$\sqrt{x}$ is real if $x \geq 0$

(If the number under the root is 0 or positive, then the square root of that number is a **real** number).

$\sqrt{x}$ is non-real if $x < 0$

(If the number under the root is negative, then the square root of that number is a **non-real** number).

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Example: For which values of x is the expression real?

$$\sqrt{\frac{x+3}{4-x}}$$

The following rules and laws must be learnt and applied:

- $a^m.a^n = a^{m+n}$

  The bases are the same so you will then add the exponents

  For example

  $x^4.x = x^4.x^1 = x^{4+1} = x^5$ (same bases, add exponents)

  $2^3.2^3 = 2^6 = 64$ (same bases, add exponents)

- $\frac{a^m}{a^n} = a^{m-n}$

  The bases are the same so you will then subtract the exponents

  For example

  $\frac{x^4}{x} = \frac{x^4}{x^1} = x^{4-1} = x^3$ (same base, subtract exponents)

  $\frac{2^{12}}{2^{10}} = 2^{12-10} = 2^2 = 4$ (same base, subtract exponents)

- $(a^m)^n = a^{m\times n}$

  With this rule, you will need to multiply the exponents

  For example

  $(3x^4)^2 = (3^1x^4)^2 = 3^{1\times 2}.x^{4\times 2} = 9x^8$

- $x^{-n} = \frac{1}{x^n}$ **and** $\frac{1}{x^{-n}} = x^n$

  With this rule, you make the exponents positive

  For example

  $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\qquad$ $\frac{1}{3^{-2}} = 3^2 = 9$

  $a^0 = 1$

  Any number raised to an exponent of 0 equals 1

  For example

  $3^0 = 1$

  $4x^0 = 4\times 1 = 4$

- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

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For example

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{16}{9}$$

Rules for rational exponents and surds

- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

  For example

  $2^{\frac{3}{4}} = \sqrt[4]{2^3}$

- $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

  For example

  $\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{4 \times 2} = \sqrt[3]{8} = 2$

- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

  For example

  $$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2} = 2\frac{1}{2}$$

X-ample Questions

Question 1

Show all working out. *(Apply all the rules separately and work with positive exponents)*

$5^3 . 5^4$	
$(2x^3y^2) . (-3x^5y^4)$	
$\left(\frac{12x^3}{15y^2}\right)^3$	
$\left(\frac{1}{2^{-2}} - \frac{1}{3^{-2}}\right)^{-2}$	

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Question 2

Simplify the following:

a. $\sqrt{\frac{15^x.3^x}{9^{x+1}.5^{x-2}}}$

b. $\frac{3^{2+x}-4.3^x}{2.3^{x-1}+3^x}$

c. $\frac{9^{n-1}.27^{3-2n}}{81^{2-n}}$

Question 3

Simplify:

a. $\sqrt{12}+4\sqrt{75}$
b. $(2+3\sqrt{2})^2$
c. $\frac{y}{\sqrt{x}+2}$
d. $\frac{x-16}{\sqrt{x}+4}$

Question 4: (FHSST Grade 11 Mathematics)

A type of bacteria has a very high exponential growth rate at 80% every hour. If there are 10 bacteria, determine how many there will be in 5 hours, in 1 day and in 1 week?

X-ercises

Question 1

For which values of x is the expression real?

$$\sqrt{\frac{5+x}{x+1}}$$

Question 2

A species of extremely rare, deep water fish has an extremely long lifespan and rarely have children. If there are a total 821 of this type of fish and their growth rate is 2% each month, how many will there be in half of a year? What will be the population be in 10 years and in 100 years?

Question 3

Study the answers given to the questions below and describe the error made and provide the correct answer.

Attempt at the Question	Errors made	Correct answer
1. $-7x^{-3} = \frac{1}{7x^3}$		
2. $\frac{6^{2x}}{2^x.3^{x+1}} = \frac{6^{2x}}{6^{2x+1}} = \frac{6^{2x}}{6^{2x}.6^1} = \frac{1}{6}$		
3. $\frac{2.3^x - 3^{x-2}}{3^x + 3^{x-2}} = \frac{3^x\left(2-3^{-2}\right)}{3^x\left(3^{-2}\right)} = \frac{\left(2-3^{-2}\right)}{3^{-2}} = \frac{3^2}{2-3^2} = \frac{9}{-7}$		
4. $\sqrt{18}+\sqrt{32} = \sqrt{50} = \sqrt{25.2} = 5\sqrt{2}$		
5. $\left(\sqrt{5}-2\sqrt{3}\right)^2 = \left(\sqrt{5}\right)^2 - \left(2\sqrt{3}\right)^2 = 5-2.3 = -1$		
6. $4^{-\frac{1}{2}} = \frac{1}{4^2} = \frac{1}{16}$		