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SESSION 15: PROBABILITY

KEY CONCEPTS:

- Compare the relative frequency of an experimental outcome with the theoretical
- probability of the outcome.
- Use Venn diagrams as an aid to solving probability problems.
- Mutually exclusive events
- Complementary events.

TERMINOLOGY

Experiment: an uncertain process.

An outcome of an experiment: a single result of that experiment.

The sample space of an experiment: the set of all possible outcomes of that experiment.
The sample space is denoted with the symbol S and the size of the sample space (the total number of possible outcomes) is denoted with n(S).

An event: a specific set of outcomes of an experiment that you are interested in. An event is denoted with the letter E and the number of outcomes in the event with n(E).

A probability: a real number between 0 and 1 that describes how likely it is that an event will occur.

The relative frequency of an event: the number of times that the event occurs during experimental trials, divided by the total number of trials conducted.

The union of two sets: a new set that contains all of the elements that are in at least one of the two sets. The union is written as A U B.

The intersection of two sets: a new set that contains all of the elements that are in both sets. The intersection is written as A ∩ B.

Mutually exclusive events: two events that cannot occur at the same time. Whenever
an outcome of an experiment is in the first event, it can not also be in the second event.

The complement of a set A: a different set that contains all of the elements that are not in A. We write the complement of A as A^{l}, or as “not(A)”.

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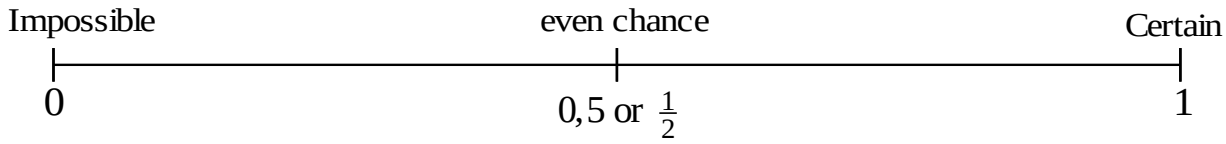
X-PLANATION

The probability scale



We write probabilities as fractions, decimals or percentages. The less likely an event is to happen, the smaller the fraction. The more likely the probability, the greater the fraction.

Calculating probabilities

When all outcomes of an activity are equally likely, you can calculate the probability of an event happening by using the following definition:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} = \frac{n(E)}{n(S)}$$

Inclusive events
Events which do have elements in common are called inclusive events. These events can happen at the same time or simultaneously.
For **inclusive** events, the following rules are true:
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Alternative notation: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Mutually exclusive events
Events which have no elements in common are called mutually exclusive events. These events cannot happen at the same time or simultaneously.
For two **mutually exclusive** events A and B, the following rules are true:
$P(A \text{ or } B) = P(A) + P(B)$ where $P(A \text{ and } B) = 0$
Alternative notation: $P(A \cup B) = P(A) + P(B)$ where $P(A \cap B) = 0$

Exhaustive events
Two events A and B are said to be exhaustive if, together, they cover all elements of the sample space, i.e. $P(A \text{ or } B) = 1$.

Complementary events
Mutually exclusive, exhaustive events are called complementary events.
For any two **complementary events**: $P(\text{not } A) = 1 - P(A)$
Alternative notation: $P(A^{|}) = 1 - P(A)$

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X-AMPLE QUESTIONS:

Question 1:

There are 12 white marbles in a box. Each marble is numbered from 1 to 12. A marble is drawn out of the box at random. Calculate the probability of drawing:

(a) a marble numbered 4. (1)

(b) an odd numbered marble. (2)

(c) a prime numbered marble. (1)

(d) a prime numbered marble or a marble whose number is a multiple of 4. (3)

(e) a marble numbered 13. (1)

(f) a marble whose number is a natural number. (1)

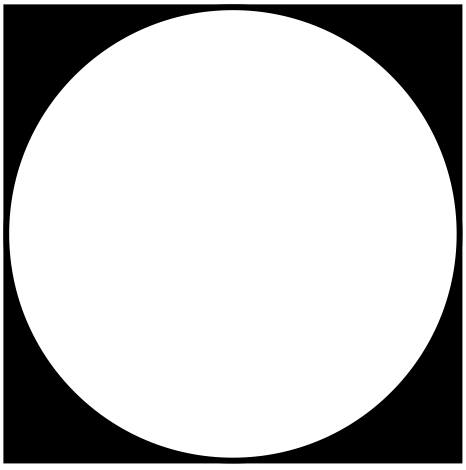
QUESTION 2

A card is drawn from a pack of 52 cards (joker excluded). Determine the probability of drawing:

(a) a diamond. (2)
(b) a jack of clubs (1)
(c) an ace. (2)
(d) a king or queen. (2)
(e) neither a heart or a spade. (2)

QUESTION 3

A circle is inscribed in a square of length 50cm. Calculate the probability of choosing a point inside the shaded area rounded off to four decimal places. (4)



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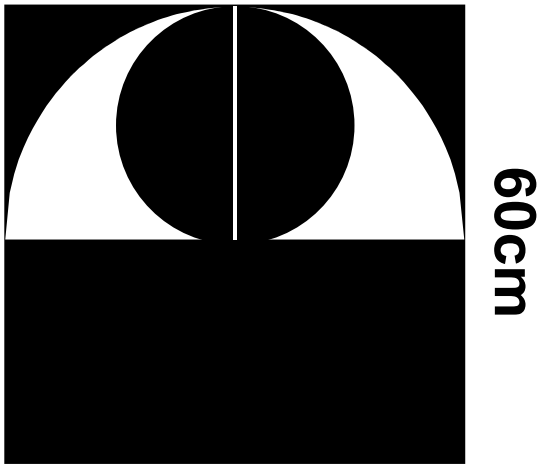
QUESTION 4

The length of the square is 60cm. A semi-circle is inscribed in the square and a smaller circle is drawn inside the semi-circle. The smaller circle's diameter is the radius of the semi-circle. Calculate the probability of choosing a point inside the non-shaded, white area rounded off to three decimal places.

(5)



QUESTION 5

There are 12 white marbles in a box. Each marble is numbered from 1 to 12. A marble is drawn out of the box at random. Calculate the probability of drawing:

(a) a marble numbered 7. (1)

(b) an even numbered marble. (2)

(c) a marble whose number is not prime. (1)

(d) a prime numbered marble or a marble whose number is a multiple of 6. (3)

(e) a marble numbered 18. (1)

(f) a marble whose number is an integer. (1)

QUESTION 6

There are 12 white marbles in a box. Each marble is numbered from 1 to 12. A marble is drawn out of the box at random. The following events are given:

$A = \{\text{drawing an even number}\}$

$B = \{\text{drawing an odd number}\}$

$C = \{\text{drawing a number greater than 7}\}$

$D = \{\text{drawing a number less than 5}\}$

$E = \{\text{drawing natural numbers less than 7}\}$

$F = \{\text{drawing natural numbers greater than 4}\}$

(a) Draw a Venn diagram to represent events A and B. (3)
(b) Determine P(A or B) (1)
(c) Determine P(A and B) (1)
(d) Show by using the rules that A and B are mutually exclusive and exhaustive. (2)
(e) Are events A and B complementary? Verify your answer by using the rule for complementary events. (2)
(f) Draw a Venn diagram to represent events A and C. (3)
(g) Determine P(A or C) (1)
(h) Determine P(A and C) (1)
(i) Show by using the rules that A and C are inclusive but not exhaustive. (7)
(j) Are events A and C complementary? Verify your answer by using the rule for complementary events. (2)
(k) Draw a Venn diagram to represent events C and D.
(l) Determine by using the rules, whether C and D are mutually exclusive or inclusive, exhaustive or not exhaustive, complementary or not complementary.
(m) Draw a Venn diagram to represent events E and F.
(n) Determine by using the rules, whether E and F are mutually exclusive or inclusive, exhaustive or not exhaustive, complementary or not complementary.

QUESTION 7

G and H are inclusive events in a sample space S. If it is given that $P(G \text{ or } H) = \frac{3}{4}$, $P(G) = \frac{2}{5}$ and $P(H) = \frac{1}{2}$, determine:

(a) P(G and H) if $n(S) = 20$.
(b) P(not G)

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QUESTION 8:
All the clubs are taken out of a pack of cards. The remaining cards are then shuffled and one card chosen. After being chosen, the card is replaced before the next card is chosen.

(a) What is the sample space?
(b) Find a set to represent the event, P, of drawing a picture card.
(c) Find a set for the event, N, of drawing a numbered card.
(d) Represent the above events in a Venn diagram.
(e) What description of the sets P and N is suitable?

X-ercise
A letter is drawn from the word PROBABILITY. Find the probability of:
(a) drawing the letter P.
(b) drawing the letter I.
(c) drawing the letter A.
(d) drawing a vowel.
(e) drawing the letter B.
(f) not drawing a vowel.