SESSION 9: FUNCTIONS

KEY CONCEPTS:

- Definitions & Terminology
- Graphs of Functions
  - Straight line
  - Parabola
  - Hyperbola
  - Exponential
- Sketching graphs
- Finding Equations
- Combinations of graphs

TERMINOLOGY

A function: A mathematical relationship between two variables (x and y), where every input value (usually x) has one output value (usually y)

The domain of a function: The set of all independent x-values for which there is one dependent y-value according to that function.

The range of a function: The set of all dependent y-values which can be obtained using an independent x-value.

The y-intercept: The y value when $x=0$. The point on the graph where the line or curve cuts the y-axis.

The x-intercept: The x-values when $y=0$. The point or points on the graph where the line or curve cuts the x-axis.

X-PLANATION

Set Notation

$\{x : x \in \mathbb{R}, x > 0\}$	The set of all x-values such that x is an element of the set of real numbers and is greater than 0.
$\{y : y \in \mathbb{N}, 3 < y \leq 5\}$	The set of all y-values such that y is a natural number, is greater than 3 and is less than or equal to 5.
$\{z : z \in \mathbb{Z}, z \leq 100\}$	The set of all z-values such that z is an integer and is less than or equal to 100.

Interval Notation

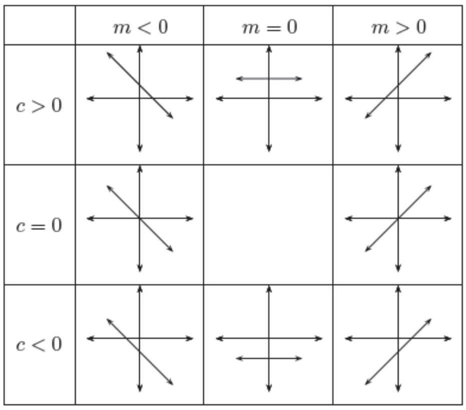
$(3; 11)$	Round brackets indicate that the number is not included. This interval includes all real numbers greater than but not equal to 3 and less than but not equal to 11.
$(-\infty; -2)$	Round brackets are always used for positive and negative infinity. This interval includes all real numbers less than, but not equal to -2.
$[1; 9)$	A square bracket indicates that the number is included. This interval includes all real numbers greater than or equal to 1 and less than but not equal to 9.

Straight line: $y = mx + c$ or $y = ax + q$

We notice that the value of *a* or *m* affects the slope of the graph. As *a* or *m* increases, the gradient of the graph increases. If *a* or *m* > 0 then the graph increases from left to right (slopes upwards). If *a* or *m* < 0 then the graph increases from right to left (slopes downwards).

If two lines are parallel, it is because their gradients are equal. If they are perpendicular, they meet at an angle of 90˚, the gradients are the negative inverse of each other and have the product of -1.

We also notice that the value of *q* or *c* affects where the graph cuts the y-axis. For this reason, *q* or *c* is known as the y-intercept. If *q* or *c* > 0 the graph shifts vertically upwards. If *q* or *c* < 0, the graph shifts vertically downwards.



Three methods to plot straight lines are the table method, gradient intercept method and the dual intercept method.

Quadratic functions or parabolas: $y = ax^2 + q$

The effect of *q* is called a vertical shift because all points are moved the same distance in the same direction (it slides the entire graph up or down).

For *q* > 0, the graph of *f(x)* is shifted vertically upwards by *q* units. The turning point of *f(x)* is above the y-axis.

For *q* < 0, the graph of *f(x)* is shifted vertically downwards by *q* units. The turning point of f(x) is below the y-axis.

The sign of *a* determines the shape of the graph.

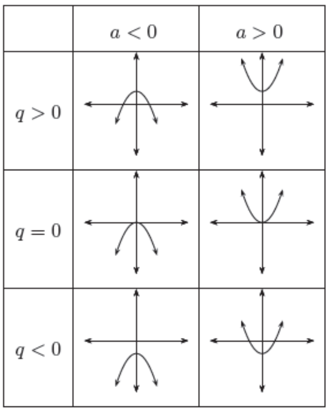
$a > 0$ (a positive smile)

For a > 0, the graph of f(x) is a "smile" and has a minimum turning point at (0; q). The graph of f(x) is stretched vertically upwards; as a gets larger, the graph gets narrower. For 0 < a < 1, as a gets closer to 0, the graph of f(x) get wider.

$a < 0$ (a negative frown)

For a < 0, the graph of f(x) is a "frown" and has a maximum turning point at (0; q). The graph of f(x) is stretched vertically downwards; as a gets smaller, the graph gets narrower. For -1 < a < 0, as a gets closer to 0, the graph of f(x) get wider.

The axis of symmetry in Grade 10 will always be the y-axis. This changes in Grade 11.



Hyperbola: $y = \frac{a}{x} + q$

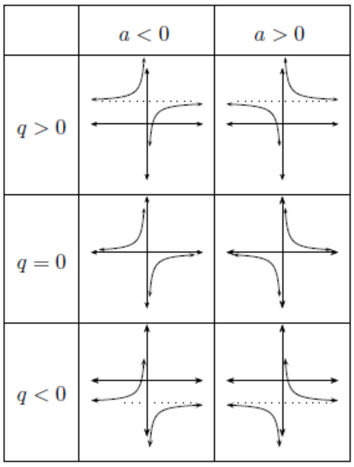
The effect of *q* is called a vertical shift because all points are moved the same distance in the same direction (it slides the entire graph up or down). For *q* > 0, the graph of *f(x)* is shifted vertically upwards by *q* units. For *q* < 0, the graph of *f(x)* is shifted vertically downwards by *q* units.

The horizontal asymptote is the line *y* = *q* and the vertical asymptote is always the y-axis, the line
x = 0.

The sign of *a* determines the shape of the graph.

If *a* > 0, the graph of *f(x)* lies in the first and third quadrants. For *a* > 1, the graph of *f(x)* will be further away from the axes than $= \frac{1}{x}$. For 0 < *a* < 1, as a tends to 0, the graph moves closer to the axes than $y = \frac{1}{x}$.

If *a* < 0, the graph of *f(x)* lies in the second and fourth quadrants. For *a* < -1, the graph of *f(x)* will be further away from the axes than $y = -\frac{1}{x}$. For -1 < *a* < 0, as a tends to 0, the graph moves closer to the axes than $y = -\frac{1}{x}$.



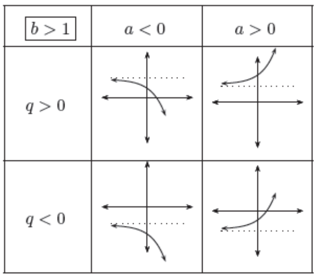
Exponential functions: $y = a.b^x + c; b \neq 1; b > 0$

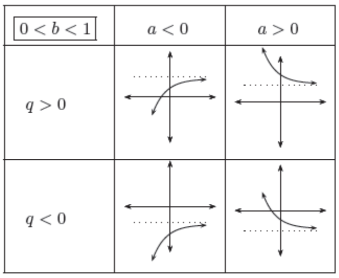
The effect of *q* is called a vertical shift because all points are moved the same distance in the same direction (it slides the entire graph up or down).

The horizontal asymptote is the line *y* = *q*.

a determines whether the graph is above or below the asymptote. For *a* > 0, the graph is above the asymptote. For *a* < 0, the graph is below the asymptote. It reflects the graph about the horizontal asymptote.

b determines the direction of the curve of the graph. If *b>1*, the graph will start close to the asymptote and curve further away from it. If *0<b<1*, the graph will start further away from the asymptote and move closer towards it.





X-AMPLE QUESTIONS:

Question 1:
Sketch the graphs of the following linear functions:

a) $y = 2x + 4$

b) $y - 3x = 0$

c) $2y = 4 - x$

Question 2:
Sketch the following quadratic functions:

a) $y = x^2 + 3$

b) $y = \frac{1}{2}x^2 + 4$

c) $y = 2x^2 - 4$

Question 3:
Sketch the following hyperbolic functions and identify the asymptotes

a) $y = \frac{3}{x} + 4$

b) $y = \frac{1}{x}$

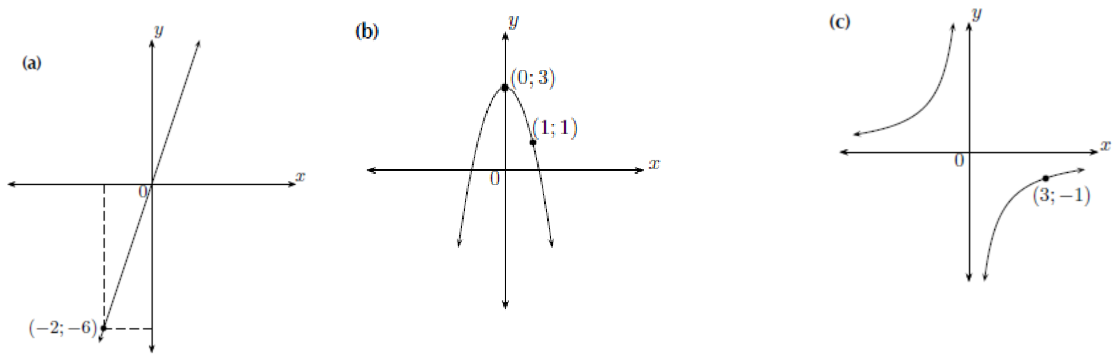
c) $y = \frac{2}{x} - 2$

Question 4:
Sketch the following exponential functions and identify the asymptotes:

a) $y = 3^x + 2$

b) $y = -4 \times 2^x$

Question 5:
Given the general equations $y = mx + c,\ y = ax^2 + q,\ y = \frac{a}{x} + q$, and $y = b^x + q$, determine the specific equations for each of the following graphs:



Question 6:

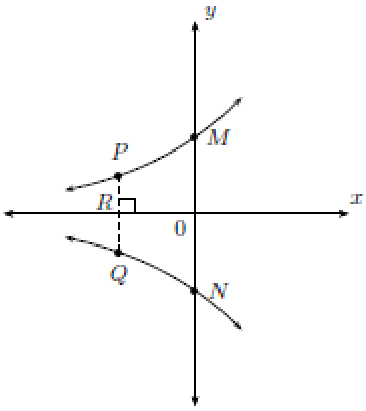
Given the functions $f(x) = 2x^2 - 6$ and $g(x) = -2x + 6$:

a) Draw *f* and *g* on the same set of axes.

b) Calculate the points of intersection of *f* and *g.*

X-ercises

1. $y = 2^x$ and $y = -2^x$ are sketched below. Answer the questions that follow:



a) Calculate the coordinates of *M* and *N.*

b) Calculate the length of *MN*.

c) Calculate length *PQ* if *OR* = 1 unit.

d) Give the equation of $y = 2^x$ reflected about the y-axis.

e) Give the range of both graphs.

Solution

a.) $M(0;1)$ $N(0;-1)$

b.) MN = 2

c.) $y = \left(\frac{1}{2}\right)^x$

d.) $range\ of\ y = 2^x\ is\quad 0 < y$

$range\ of\ y = -2^x\ is\quad 0 > y$