SESSION 2: EXPONENTS

KEY CONCEPTS:

- Simplify expressions using the laws of exponents for rational exponents.
- Solving exponential equations

X-PLANATION

Exponential notation is a short way of writing the same number multiplied by itself many times. For any real number a and natural number n, we can write a multiplied by itself n times as a^n. a is called the base, n is called the exponent or index.

When doing calculations with exponents we use the following Law of Exponents

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $(a^m)^n = a^{mn}$

where $a > 0$, $b > 0$ and $m, n \in \mathbb{Z}$.

Simplifying Exponents

Changing the bases to products of their prime factors

Question:

Simplify: $\frac{2^{2n} \times 4^{n} \times 2}{16^{n}}$

Solution:

Step 1: ***Change the bases to prime numbers***

$$\frac{2^{2n} \times 4^{n} \times 2}{16^{n}} = \frac{2^{2n} \times (2^{2})^{n} \times 2^{1}}{(2^{4})^{n}}$$

Step 2: ***Simplify the exponents***

$$= \frac{2^{2n} \times (2^{2})^{n} \times 2^{1}}{(2^{4})^{n}}$$

$$= \frac{2^{2n + 2n + 1}}{2^{4n}}$$

$$= \frac{2^{4n + 1}}{2^{4n}}$$

$$= 2^{4n + 1 - (4n)}$$

$$= 2$$

Factorising: Common Factor

Question:

Simplify: $\dfrac{2^t - 2^{t-2}}{3 \times 2^t - 2^t}$

Solution:

Step 1: ***Simplify to a form that can be factorised***

$$\frac{2^t - 2^{t-2}}{3 \times 2^t - 2^t} = \frac{2^t - (2^t \times 2^{-2})}{2^t(3 - 1)}$$

Step 2: ***Take out a common factor***

$$= \frac{2^t(1 - 2^{-2})}{2^t(3 - 1)}$$

Step 3: ***Cancel the common factor and simplify***

$$= \frac{1 - \frac{1}{4}}{2}$$

$$= \frac{\frac{3}{4}}{2}$$

$$= \frac{3}{8}$$

Factorising: Difference of two squares

Question:

Simplify: $\frac{9^x - 1}{3^x + 1}$

Solution:

Step 1: ***Change the bases to prime numbers***

$$\frac{9^x - 1}{3^x + 1} = \frac{(3^2)^x - 1}{3^x + 1}$$

$$= \frac{(3^x)^2 - 1}{3^x + 1}$$

Step 2: ***Factorise using the difference of squares***

$$= \frac{(3^x - 1)(3^x + 1)}{(3^x + 1)}$$

Step 3: ***Simplify***

$$= 3^x - 1$$

Simplifying rational exponents (fractions) without the use of the calculator:

1. $t^{\frac{1}{4}} \times 3t^{\frac{7}{4}}$
   $= 3t^{\frac{1}{4}+\frac{7}{4}}$
   $= 3t^2$

2. $(0{,}25)^{\frac{1}{2}}$
   $= \left(\frac{25}{100}\right)^{\frac{1}{2}}$
   $= \left(\frac{5^2}{10^2}\right)^{\frac{1}{2}}$
   $= \left(\left(\frac{5}{10}\right)^2\right)^{\frac{1}{2}}$
   $= \frac{1}{2}$

Solving exponential equations

Question:

Solve for x: $3^{x+1} = 9$.

Solution:

Step 1: ***Change the bases to prime numbers***

$3^{x+1} = 3^2$

Step 2: ***The bases are the same so we can equate exponents***

$x + 1 = 2$
$\therefore x = 1$

Question:

Solve for t: $5^t + 3 \times 5^{t+1} = 400$.

Solution:

Step 1: ***Rewrite the expression***

$5^t + 3(5^t \times 5) = 400$

Step 2: ***Take out a common factor***

$5^t(1 + 15) = 400$

Step 3: ***Simplify***

$5^t(16) = 400$

Step 4: ***Change the bases to prime numbers***

$5^t = 5^2$

Step 5: ***The bases are the same so we can equate exponents***

$\therefore t = 2$

Question:

Solve: $p - 13p^{\frac{1}{2}} + 36 = 0$.

Solution:

Step 1: ***We notice that*** $(p^{\frac{1}{2}})2 = p$ ***so we can rewrite the equation as***

$(p^{\frac{1}{2}})^2 - 13p^{\frac{1}{2}} + 36 = 0$

Step 2: ***Factorise as a trinomial***

$(p^{\frac{1}{2}} - 9)(p^{\frac{1}{2}} - 4) = 0$

Step 3: ***Solve to find both roots***

$p^{\frac{1}{2}} - 9 = 0$	$p^{\frac{1}{2}} - 4 = 0$
$p^{\frac{1}{2}} = 9$	$p^{\frac{1}{2}} = 4$
$(p^{\frac{1}{2}})^2 = (9)^2$	$(p^{\frac{1}{2}})^2 = (4)^2$
$p = 81$	$p = 16$

Therefore $p = 81$ or $p = 16$

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X-AMPLE QUESTIONS:

Question 1:
Simplify the following exponents

1.1 $5^{2x+y}.5^{3(x+z)}$

1.2 $(b^{k+1})^{k}$

1.3. $\frac{6^{5p}}{9^{p}}$

1.4 $m^{-2t} \times (3m^{t})^{3}$

1.5 $\frac{3x^{-3}}{(3x)^{2}}$

6. $\dfrac{3^{n}.9^{n-3}}{27^{n-1}}$

7. $\left(\dfrac{2x^{2a}}{y^{-b}}\right)^{3}$

8. $\dfrac{6^{2x}.11^{2x}}{22^{2x-1}.3^{2x}}$

9. $(3^{-1} + 2^{-1})^{-1}$

10. $\dfrac{3^{t+3} + 3^{t}}{2 \times 3t}$

11. $\dfrac{2^{3p} + 1}{2^{p} + 1}$

12. $\dfrac{3^{x+1} - 3^{x}}{3^{x+2} - 2.3^{x}}$

13. $\dfrac{2^{x+1} + 2^{x-3}}{2^{x-3} + 2^{x}}$

Question 2

Solve the following exponential equations:

2.1 $3^x = \frac{1}{27}$

2.2 $5^{t-1} = 1$

2.3 $2 \times 7^{3x} = 98$

4. $2^{m+1} = (0,5)^{m-2}$

5. $m^0 + m^{-1} = 0$

6. $3^p + 3^p + 3^p = 27$

Question 3
Simplify the following expression without the use of a calculator

$$\frac{(\sqrt{2})^3 . 2^{18}}{(2^8)^2 . \sqrt{2}}$$

Question 4
Simplify the following expression without the use of a calculator

$$\left(\frac{11x^7 + x^7}{8(x^{-3})^2}\right)^{-2}$$